MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCES

SMA 230: VECTOR ANALYSIS

DATE: 15/12/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS: Answer Question one (compulsory) any other two Questions

QUESTION ONE (30 MARKS)

a) Determine the constant a so that the following vectors is solenoidal, $V = (-4x-6y+3z)i+(-2x+y-5z)j+(5x+6y+az)k$ (3 marks)

b) Find p such that the vectors $a = 2i - j + k$; $b = i + 2j - 3k$ and $c = -j + 4k$ are co-planar (3 marks)

c) If $\phi = x^2yz - 4xyz^2$. Find the directional derivative of ϕ at $P(1,3,1)$ in the direction of $2i - j - 2k$ (4 marks)

d) Find the area of triangle ABC whose vertices are $A(0,0,0)$, $B(1,1,1)$ and $C(0,2,3)$ (4 marks)

e) Given that $\phi(x,y,z) = xy^2z$ and $A = xi + j + xyk$. Find $\dfrac{\partial^3}{\partial x^2 \partial z}(\phi A)$ at the point $P(1,2,2)$ (5 marks)

f) Find the work done in moving a particle in the force field given by $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1$, $y = 2t^2$ and $z = t^3$ from $t = 1$ and $t = 2$ (5 marks)

g) Evaluate $\iint_S F \cdot n\,dS$, where $F = 4xzi - y^2j + yzk$ and S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$ (6 marks)

QUESTION TWO (20 MARKS)

a) Given that $F = xyi - zj + x^2k$ and C is the curve $x = t^2$, $y = 2t$, $z = t^3$ from $t = 0$ and $t = 1$. Evaluate the line integrals $\int_C F \times dr$ (6 marks)

b) The acceleration of a particle at any time $t \geq 0$ is given by $a = \frac{dr}{dt} = (25\cos 2t)i + (16\sin 2t)j + 9tk$. If the velocity v and the displacement r are zero when $t = 0$. Find v and r at any time (6 marks)

c) Find a set of vectors reciprocal to the set of $2i + 3j - k$, $i - j - 2k$, and $-i + 2j + 2k$

(8 marks)

QUESTION THREE (20 MARKS)

a) Given $R(u) = 3i + (u^3 + 4u^7)j + uk$. Find $\int_1^2 R(u)\,du$ (4 marks)

b) If $F = 2zi - xj + yk$, evaluate $\iiint_V F\,dv$ where v is the region bounded by the surfaces $x = 0$, $y = 0$, $x = 2$, $y = 4$, $z = x^2$, $z = 2$ (6 marks)

c) The vector $F = (2xy + z^3)i + x^2j + 3xz^2k$

i. Show that it is a conservative force field (3 marks)

ii. find the scalar potential (5 marks)

iii. find the work done in moving an object in this field from $(1,-2,1)$ to $(3,1,4)$

(2 marks)

QUESTION FOUR (20 MARKS)

a) A particle is displaced from point $A(2,-3,1)$ to $B(4,2,1)$ under the action of constant forces $12i - 5j + 6k$; $i + 2j - 2k$ and $2i + 8j + k$. Find the work done by the forces on the particle

(4 marks)

b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$ (6 marks)

c) Evaluate $\iint_S A.n\,dS$, where $A = 18zi - 12j + 3yk$ and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant (10 marks)

QUESTIONS FIVE (20 MARKS)

a) Find the moment about the point $A(5,-1,3)$ of a force $4i+2j+k$ through point $B(5,2,4)$ (3 marks)

b) Use Divergence theorem to evaluate $\iint_S F \cdot dS$ where $F = 4xi - 2y^2 j + z^2 k$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$ (8 marks)

c) Use Stokes's theorem to evaluate $\int_C v \cdot dr$ where $v = y^2 i + xyj + xzk$ and C is the bounding curve of the hemisphere $x^2 + y^2 + z^2 = 9$ oriented in the positive direction. (9 marks)