MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SMA 465: INTRODUCTION TO MEASURE AND PROBABILITY

DATE: **TIME:**

INSTRUCTIONS TO CANDIDATES:

Answer Question One and Any Other Two Questions

QUESTION ONE (compulsory) (30 Marks)

a) Define the following terms as used in measure theory and probability

i) Interval (1 mark)

ii) Length of an integer (1 mark)

iii) Area (1 mark)

iv) Volume (1 mark)

v) Algebra (1 mark)

vi) Sigma algebra (1 mark)

b) State the fundamental theorem of Lebesque measure on $\mathbb{R}^n$ (3 marks)

c) Give the two axioms of measure space (2 marks)

d) Show that if $\alpha, \beta \epsilon\ P$ then μ (β) = μ ($\alpha \cap \beta$) + μ ($\beta \backslash \alpha$) (2 marks)

e) Describe briefly the fundamental theorems of integral functions in measure theory. (2 marks)

f) Give any two application of measure (2 marks)

g) How is an algebra distinct from a σ algebra (4 marks)

h) Differentiate between the following;

i) measurable sets and measurable functions (2 marks)

ii) measurable space and measure space (2 marks)

iii) probability space and probability measure (2 marks)

QUESTION TWO (20 MARKS)

a) State and proof the law of large numbers (3 marks)
b) Define the conditional expectation of random variables (3 marks)
c) State the properties of conditional expectation (2 marks)
d) Let A and B be two independent random variables, show that $E(A/B) = E(B)$ (3 marks)
e) Let a,b and c be jointly distributed with pdf,

$$f(a,b,c) = \begin{cases} \frac{1}{(2\pi)^{3/2}}\, e^{-\frac{1}{2}(a^2\, b^2\, c^2)} & -\infty < a,b,c < \infty \\ 0\ else\ where \end{cases}$$

Find;

i) f(a/bc), (3 marks)

ii) f(b/ac) , (3 marks)

iii) f(c/ab) (3 marks)

QUESTION THREE (20 MARKS)

a) State and explain the two theorems of integration (4 marks)
b) Define a Lebesgue integral (2 marks)
c) Explain the meaning of a Diral measure giving an example (2 marks)
d) State the central limit theorem and its importance in sampling distributions (2 marks)
e) Prove the Fatou's Lemma (5 marks)
f) Enumerate the properties of integrable functions (5 marks)

QUESTION FOUR (20 marks)

a) Define the characteristic function of a random variable (1 marks)

b) Let $\phi(t) = E[e^{itx}]$, show that;

i. E(x)=$\phi'_x(0)$ (2 marks)

ii. Var(x) =$\frac{\phi''_x(0)-(\phi'_x(0))^2}{i^2}$ (4 marks)

c) Given that $f(x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & x = 0,1,\ldots \\ 0\ elsewhere \end{cases}$

i) obtain the characteristic function of x; (8 marks)

ii) Using the characteristic function, find the mean of x (2 marks)

iii) Hence find the variance of x (3 marks)

QUESTION FIVE (20 marks)

a) Define μ – finite measure (3 marks)
b) Describe Lebesgue measure (3 marks)
c) What is an integral function (2 marks)
d) Explain three examples of measure (6 marks)
e) Describe the two modes of convergence theorems (6 marks)