MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR

MASTER OF SCIENCE (MATHEMATICAL MODELLING AND COMPUTATIONAL)

SMA 803: FOUNDATIONS OF NUMERICAL COMPUTING

DATE: 24/6/2021 **TIME: 2:00 – 5:00 PM**

INSTRUCTIONS: Answer any THREE questions

QUESTION ONE (20 MARKS)

a) Consider the following matrix

$$A = \begin{bmatrix} -4 & 14 & 0 \\ -5 & 13 & 0 \\ -1 & 0 & 2 \end{bmatrix}.$$

State the MATLAB function that does the following:

i) Computes the inverse of matrix A (1 mark)

ii) Determines the all the eigenvalues of A (1 mark)

iii) Arranges the eigenvalues of A along the leading diagonal of a 3x3 matrix. (1 mark)

b) The following MATLAB code segment implements the Gauss-Siedel iteration method to solve a 4x4 linear system of equations specified in the given lines of the code.

```
1)A = [4 2 0 0;2 8 2 0;0 2 8 2;0 0 2 4];
2)b = [4;0;0;0];
3)tol = 1.0e-9;
4)format long;
5)xguess = [1 -0.5 0.1 -0.2];
6)[x,iter] = GAUSSD(A,b,xguess,tol)
```

Complete the code by writing the missing segment i.e. the function `GAUSSD(A,b,xguess,tol)`. (7 marks)

c) Given the function $f(x)=\dfrac{x\cos x}{(x^2+1)(x+2)}$ where $x\in[0,1]$. Write a MATLAB code using `for...end` loop in steps of 0.1 to calculate the values of $f(x)$ within the `for...end` loop (5 marks)

d) Implement suitable nested loop and conditional statements to write a code segment that specifies the function below and displays it in MATLAB (5 marks)

$$f(x)=\begin{cases} x^2 & 3\le x<5 \\ 1 & x=5 \\ x & 5<x\le 7 \end{cases}$$

QUESTION TWO (20 MARKS)

a) Complete the following MATLAB code to implement the Newton-Raphson method to determine the root of the equation $-x^2+1.8x+2.5=0$, with $x_0=5$ and correctly compute the results (5 marks)

```
1) function rootNR()
2) clear all; clc;
3) x0=5;imax=15;es=.05;
4) xr=x0;iter=0;
5) while(iter <imax)
6)
7)
8)
9) if(xr~=0)
10)   ea=(abs(xr-xrold)./xr)*100;
11)   end
12)   if(ea<es)
13)   break
14)   end
15)   end
16)
17)
18)
19)   function fp=fxn2(x)
20)   fp=-2*x+1.8;
21)   end
22) end
```

b) Implement the `fprintf` built-in function in the completed code of a) above to display the iteration step, iterate x, and approximate error ea (5 marks)

c) Consider the symmetric, tridiagonal equations

$$\begin{aligned} 4x_1-x_2&=9 \\ -x_{i-1}+4x_i-x_{i+1}&=5, \quad i=2,...,7 \\ -x_7+4x_8&=5 \end{aligned}$$

Write a MATLAB code that:

i) Specifies the resulting tridiagonal matrix of coefficients (4 marks)

ii) Solves the above system by QR decomposition of the tridiagonal matrix using the code from i) above. (6 marks)

QUESTION THREE (20 MARKS)

a) The displacement formulation for a certain mass-spring system is given by the following equations of the masses:

$$\begin{bmatrix} k_1+k_2+k_3+k_5 & -k_3 & -k_5 \\ -k_3 & k_3+k_4 & -k_4 \\ -k_5 & -k_4 & k_4+k_5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}$$

where k_i , W_i , x_i are the spring stiffness, weights of masses, and displacements of the masses from the non-deformed configuration of the system. Write a MATLAB program that solves these equations by matrix inversion method. Use

$k_1 = k_3 = k_4 = 5, k_2 = k_5 = 10$

$W_1 = W_3 = 100, W_2 = 50$ (8 marks)

b) Create a MATLAB program to determine the area of the triangle with vertices at $A(1,3,2)$, $B(2,-1,1)$, $C(-1,2,3)$ (8 marks)

c) Write a MATLAB code that solves the equations $A\vec{x} = \vec{b}$ by LU decomposition method, where

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -4 & 11 & -7 \\ 4 & -7 & 6 \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 28 \\ -40 \\ 33 \end{bmatrix}$$ (4 marks)

QUESTION FOUR (20 MARKS)

a) The given nonlinear equations describe the concentrations of two reactants and one product in a certain chemical process

$$x'(t) = a(u - x(t)) - bx(t)y(t)$$
$$y'(t) = a(v - y(t)) - bx(t)y(t) \text{ where } a = 5, b = 2, u = v = 5$$
$$z'(t) = -az(t) + bx(t)y(t)$$

Write a MATLAB code that uses the modified Euler method to solve this equation for $0 \le t \le 1$ and to plot solutions $x(t), y(t), z(t)$ in same figure. Let the initial conditions be $x(0) = 1,\ y(0) = 2,\ z(0) = 3$. (14 marks)

b) Use a for-end loop in writing a function named `sum_Series` in MATLAB to calculate the sum of the first n terms of the series $\sum_{k=1}^{n} \frac{(-1)^k k}{2^k}$ for $n = 20$ (6 marks)

QUESTION FIVE (20 MARKS)

a) Use the MATLAB functions `linspace` and `surf` to visualize the surface $z = -xye^{\frac{-(x^2+y^2)}{2}}$ for $-1 \le x \le 1, -1 \le y \le 1$. (6 marks)

b) An economist models the demand for a particular commodity by the function $p = D(q) = \frac{100}{q^2 + q + 1}$ where q hundred units are sold when the price is p dollars per unit.

Create a MATLAB function that uses the Simpson's one-third rule with $n = 6$ to estimate the consumers' surplus (CS) when the level of production is $q_0 = 5$. CS is calculated as

$$CS = \int_0^{q_0} D(q)dq - p_0 q_0$$ (14 marks)