



# MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 103: ANALYTICAL GEOMETRY

DATE: 25/3/2021

TIME: 8.30-10.30 AM

## INSTRUCTIONS

Attempt question *one (compulsory)* and any other *two questions*.

### QUESTION ONE (30 MARKS)

- a) A point  $R(x, y)$  divides the line through the points  $A(4, -9)$  and  $B(-5, 25)$  externally in the ratio  $3:2$ . Calculate the coordinates of  $R(x, y)$  (3 marks)
- b) Calculate the equation of the straight line which passes through the point of intersection of the lines  $4x - 3y + 19 = 0$ ,  $12x - 5y + 3 = 0$  and makes an intercept of  $+2$  on the  $x$ -axis (5 marks)
- c) Prove that  $\cos 4\theta = 8\cos^4 \theta - 8\cos^2 \theta + 1$  (4 marks)
- d) Calculate the polar equation of the circle center  $C(5, 8)$  and radius  $r = 10$  units (5 marks)
- e) A parabola  $y = ax^2 + bx + c$  passes through the points  $(1, 0)$ ,  $(2, 2)$ , and  $(3, 10)$ . Determine its equation. (5 marks)
- f) Work out the Cartesian equation for the curve  $r \cos\left(\theta - \frac{\pi}{3}\right) = 3$  (4 marks)
- g) Determine the polar equation of the ellipse  $x^2 + 9y^2 = 9$  (4 marks)

**QUESTION TWO (20MARKS)**

- a) Determine the equation of a circle through the points  $A(6,-1)$ ,  $B(-2,5)$ ,  $C(-6,-7)$ , sketch the circle (6 marks)
- b) Calculate the equation of the circle which is tangent to the x-axis and concentric with the circle  $2x^2 + 2y^2 - 11x + 6y - 8 = 0$  (6 marks)
- c) Determine the polar equation of the circle whose center has the Cartesian coordinates  $C(5,8)$  and radius  $r = 10cm$  (5 marks)
- d) Calculate the distance from the point  $(5,8)$  to the circle  $x^2 + y^2 + 2x + 6y + 3 = 0$  (3 marks)

**QUESTION THREE (20MARKS)**

- a) Solve for  $x$  in  $\sin 3x + \sin x = 0$  (4 marks)
- b) Determine the value of  $x$  for which  $2\cos x - 2\sqrt{3}\sin x = 3$  (5 marks)
- c) Show that
- i.  $\frac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = \tan 3\theta$  (4 marks)
- ii.  $\sin 4\theta = 4\cos^3 \theta \sin \theta - 4\sin^3 \theta \cos \theta$  (3 marks)
- d) Given that  $\cos 2x = \frac{1}{8}$ , calculate the value of  $\cos x$  (4 marks)

**QUESTION FOUR (20 MARKS)**

- a) Sketch the curve and label axes, foci and vertices described by the equation  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  (6 marks)
- b) Calculate the point on the parabola  $y^2 - 12y - 14x + 162 = 0$  through which the tangents which meet at  $Q(2,4)$  pass (6 marks)
- c) Determine the equation of the tangent to the parabola  $x^2 - 22x + 24y + 169 = 0$  at the point whose focal distance is 11 units (8 marks)

**QUESTION FIVE (20 MARKS)**

- a) Sketch the curve and label axes, foci and vertices described by the equation

$$\frac{x^2}{25} - \frac{y^2}{16} = -1 \quad (12 \text{ marks})$$

- b) Calculate the angle between the lines

$$y = 2x + 5 \quad \text{and} \quad y = 12x - 1 \quad (4 \text{ marks})$$

- c) Calculate the normal equation of the line with  $\rho = 10 \text{ units}$  ,  $\omega = 60^\circ$  (4 marks)