



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 302: FINANCIAL MATHEMATICS II:

DATE: 26/8/2022

TIME: 11.00-1.00 PM

INSTRUCTION:

Answer Question One and Any Other Two Questions

QUESTION ONE (30 MARKS)

- a) Define the following terms:
- i. Arbitrage. (2 marks)
 - ii. Hedging. (2 marks)
- b) Consider a fixed interest security that pays coupons of 10% at the end of each year and is redeemable at par at the end of the third year.
- Calculate (using an effective interest rate of 8% pa) the:
- i. Volatility of the cash flows. (5 marks)
 - ii. Discounted mean term of the cash flows. (1 mark)
- c) Show that
- $$E(S_n^k) = (1 + j)^n \quad (5 \text{ marks})$$
- d) A three month forward contract exists in a zero coupon corporate bond with a current price per £100 nominal of £42.60. the yield available on three month government securities of 6% p.a effective. Calculate the forward price. (5 marks)

- e) A ten year bond with half yearly coupons of 6% p.a has just been issued with a redemption yield of 9% p.a effective. It is redeemable at par. What price would an investor paying 15% tax on income pay for the bond? Tax payments are due four months after each coupon is received. (5 marks)
- f) Consider a two factor model. if :
- The mean specific return is 1.0%
 - The expected values of the two factors are 3.0% and 2.2% and ,
 - The sensitivities of investment returns to each of the factors are 0.8 and - 0.3 respectively.
- What is the expected return predicted by the model? (5 marks)

QUESTION TWO (20 MARKS)

In any year, the interest rate per annum effective on monies invested with a given bank has mean value j and standard deviation S and is independent of the interest rates in all previous years. Let S_n be the accumulated amount after n years of a single investment of 1 at time $t = 0$.

If the interest rate per annum effective, in any year, is equally likely to be i_1 , or i_2 ($i_1 > i_2$). No other values are possible.

- a) Derive expressions for j and S^2 in terms of i_1 and i_2 . (5 marks)
- b) The accumulated value at time $t = 25$ years of £1 million invested with the bank at time $t = 0$ Has expected value £5.5 million and standard deviation £0.5 million. Calculate the values of i_1 and i_2 . (15 marks)

QUESTION THREE (20 MARKS)

A fund must make payments of £50,000 at the end of the sixth and eighth year. Show that, if interest rates are currently 7% p.a at all durations, immunization to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5 year zero – coupon bond and a 10 year zero coupon bond.

QUESTION FOUR (20 MARKS)

The force of interest $\delta(t)$ is a function of time and at any time t , measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.04 & 0 < t \leq 5 \\ 0.008t & 5 < t \leq 10 \\ 0.005t + 0.0003t^2 & 10 < t \end{cases}$$

- Calculate the present value of a unit sum of money due at time $t = 12$. (13 marks)
- Calculate the effective annual rate of interest over the 12 years. (2 marks)
- Calculate the present value at time $t = 0$ of a continuous payment stream that is paid at the rate of $e^{-0.05t}$ per unit time between time $t = 2$ and time $t = 5$. (5 marks)

QUESTION FIVE (20 MARKS)

An investor has the choice of the following assets that earn rates of the return as follows in each of the four possible states of the world.

State	probability	Asset 1	Asset 2	Asset 3
1	0.2	5%	5%	6%
2	0.3	5%	12%	5%
3	0.1	5%	3%	4%
4	0.4	5%	1%	7%
Market capitalization		100,000	17,546	82,454

Determine the market price of risk assuming CAPM holds and define the terms.

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