MACHAKOS UNIVERSITY

University Examinations 2023/2024

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF MECHANICAL AND MANUFACTURING ENGINEERING

FOURTH YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING

EMM 403: SOLID AND STRUCTURAL MECHANICS 111

DATE: **TIME: 2 HOURS**

INSTRUCTIONS

This paper contains FIVE questions

Question ONE is **compulsory** and carries 30 Marks.

Questions TWO – FIVE carries 20 Marks each. **Answer any Two.**

QUESTION ONE (COMPULSORY) (30 MARKS)

a) An aluminium column is fixed at one end at the bottom and free at the top as shown in **Fig. Q1 a)**. Determine the minimum value of P if a factor of safety of 2 is desired. (Take L = 5 m, E = 70 GPa, $\sigma_y = 200$ MPa, $A = 7.5 * 10^{-3}$ m^2, $I_x = 6.13 * 10^{-5}$ m^4, $I_y = 2.32 * 10^{-5}$ m^4) **(8 Marks)**

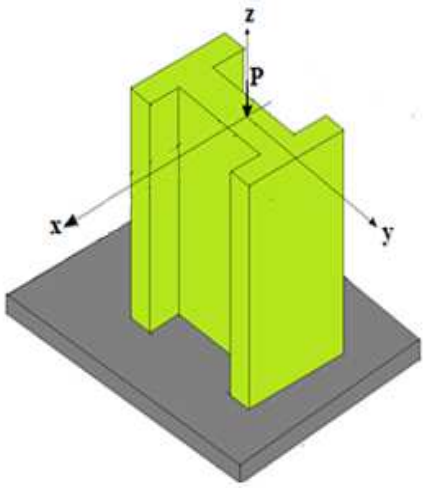

Fig. Q1 a)

b) A cylinder has an inside diameter of 150 mm and outside diameter of 450 mm. If it is subjected to an internal pressure of 160 MPa and external pressure of 80 MPa, determine radial and tangential stresses at the mean radius. **(3 Marks)**

c) In **Fig. Q1 c)**, load of P, 10MN, is supported at B by two rods of same material and diameter, 10 mm. Determine the horizontal and vertical deflections at point B using Castigiliano's theorem. Take E = 200 GPa **(9 Marks)**

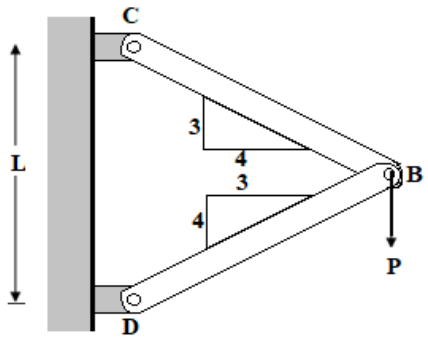

Fig. Q1 c)

d) Consider a simply supported rectangular plate of sides $a\ x\ b$ and thickness t. The plate carries a uniformly distributed load P per unit area over the entire surface. Show that the plates maximum bending stress will be given by

$$\sigma_{max} = \frac{Pa^2b^2}{2t^2\sqrt{(a^2+b^2)}}$$ **(6 Marks)**

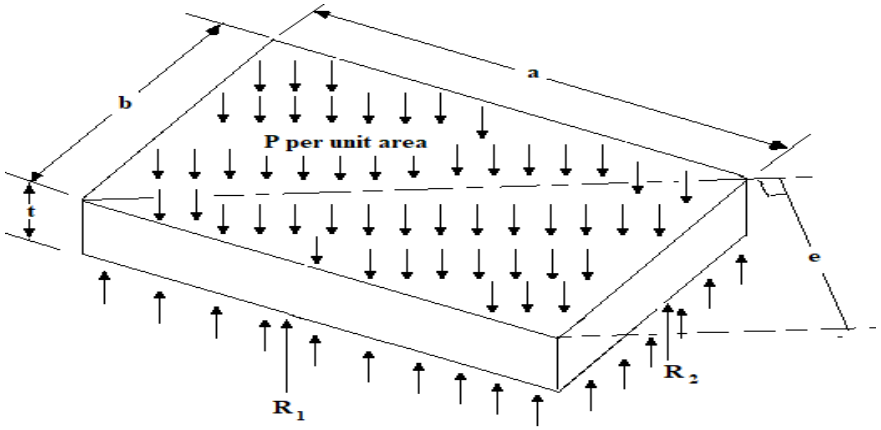

Fig. Q1 d)

e) A thin disc of mass 50 kg, radius 60 mm and thickness 25 mm rotates at a uniform angular speed of 70 rad/s. If the disc is fabricated from a material whose Poisson's ratio is 0.25, determine the radial and circumferential stresses developed at mid radius of the disc when in rotation. **(4 Marks)**

QUESTION TWO (20 MARKS)

Show that the tangential and radial stresses for a thick-walled cylinder are given by the following Lames equation

$$\sigma_r = \left(\frac{a^2p_i - b^2p_o}{b^2 - a^2}\right) - \left(\frac{a^2b^2(p_i - p_o)}{b^2 - a^2}\right)\frac{1}{r^2}$$

$$\sigma_t = \left(\frac{a^2p_i - b^2p_o}{b^2 - a^2}\right) + \left(\frac{a^2b^2(p_i - p_o)}{b^2 - a^2}\right)\frac{1}{r^2}$$

For a thick-walled pressure vessel of internal radius, a, and external radius, b, subjected to internal and external pressures of p_i and p_o respectively, derive the Lames equations for determining radial and circumferential stresses

QUESTION THREE (20 MARKS)

a) State four assumptions made in deriving the Euler's Theorem. **(4 Marks)**

b) Starting from first principles, derive Euler's equation for crippling load for the column loaded as shown in **Fig Q3 b)**. **(16 Marks)**

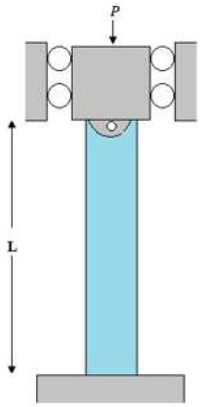

Fig. Q3 b)

QUESTION FOUR (20 MARKS)

Fig. Q 4) shows a circular plate of radius ***a*** subjected to a uniform pressure ***P*** or load ***P*** per unit area.

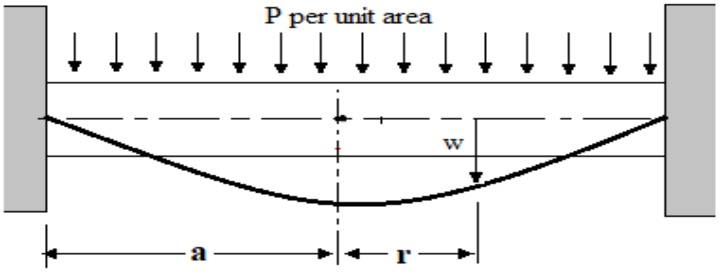

Fig. Q 4)

a) Starting from $r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{1}{D}\int_0^r Prdr$, derive an equation for deflection ($\boldsymbol{w}$) of the plate and hence determine the maximum deflection in the plate. **(15 Marks)**

b) Obtain simplified equation that can be used to determine the radial moment at any radius r in terms of ***P***, thickness ***t***, plate radius ***a*** and Poisson ratio $\boldsymbol{\nu}$. **(5 Marks)**

QUESTION FIVE (20 MARKS)

a) The general equations for determination of circumferential and radial stresses of a thin disc rotating at an angular speed of ω are;

$\sigma_\theta = A + \frac{B}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2$

$\sigma_r = A - \frac{B}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2$ where A and B are constants.

A circular disc of uniform thickness, internal radius R_i and outer raius R_o, rotates at a uniform angular speed ω. Determine expressions for maximum and mimium circumferential and radius stresses. Sketch the stresse variations in the disc. (**14 Marks)**

b) i) State one example of a rotating thin disc. **(1 Mark)**

ii) A thin disc of mass 50 kg, radius 60 mm and thickness 25 mm rotates at a uniform angular speed of 70 rad/s. If the disc is fabricated from a material whose Poisson's ratio is 0.25, determine the radial and circumferential stresses developed at mid radius of the disc when in rotation. **(5 Marks)**