MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTCS & PROGRAMMING.

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 330-NUMERICAL ANALYSIS

DATE: 25/3/2021 **TIME: 8.30-10.30 AM**

INSTRUCTION TO CANDIDATES

ANSWER **QUESTION ONE** AND **ANY TWO** OTHER QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

a) Estimate the error in z if $z = \frac{x}{y} + 2x$ given that $x = 3.213$ and $y = 6.172$ (2 marks)

b) Show that the operators μ and E commute. (3 marks)

c) Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method. (5 marks)

d) prove that the forward difference of the quotient of two functions is given by

$$\Delta\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)}$$ (5 marks)

e) Set up Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (7 marks)

f) find a root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ with an accuracy of 10^{-4}

(8 marks)

QUESTION TWO (20 MARKS)

a) If E, μ and δ denote shift, average and central difference operators, in analysis of data with equal spacing h, prove the following

$E^{1/2} = \mu + \frac{\delta}{2}$ (2 marks)

b) Starting with $x_0 = 3$, find an approximate root of $f(x) = x^3 - 3x - 5$ correct to 3dp by Newton-Raphson method upto 4 iterations (5 marks)

c) Solve the equation $x^3 = Sinx$. Considering various $\phi(x)$, discuss the convergence of the solution. (5 marks)

d) using the values given in the following table, find $Cos0.28$ by linear interpolation and by quadratic interpolation and compare the results with the value 0.96106 (exact to 5D)

(8 marks)

QUESTION THREE (20 MARKS)

a) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(4 marks)

b) Construct the forward difference table, where $f(x) = \frac{1}{x}, x = 1(0.2)2,4D$ (4 marks)

c) Find the cubic polynomial which takes the following values; $f(1) = 24,\ f(3) = 120,\ f(5) = 336,$ and $f(7) = 720.$ hence or otherwise, obtain the value of $f(8)$. (6 marks)

d) Evaluate $\int_0^1 e^{-x^2} dx$ by means of Trapezoidal rule with $n = 10$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Find the percentage error in the quotient $\frac{25.4}{12.37}$ (4 marks)

b) Find $\int_0^{10} \frac{1}{1+x^2}dx$ using Simpson's one third rule. (7 marks)

c) Find the interpolating polynomial by Newton's divided formula for the following table and then calculate $f(2.1)$.

x	0	1	2	4
$f(x)$	1	1	2	5

(9 marks)

QUESTION FIVE (20 MARKS)

a) Construct the difference table for the following data tabulated function:

x	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	1.5431	1.8107	2.1509	2.5775	3.1075	3.7622

Use suitable Newton's interpolating polynomial to estimate f(1.1) and f(1.9) (10 marks)

b) Use the equation $f(x) = 3x^3 - 6x - 2$ to show that $x_n = \sqrt[3]{\frac{6x_n + 2}{3}}$, use $x_0 = 1$ get linear iterates to 5 dp (10 marks)