MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (COMPUTER SCIENCE)

SCO 111/SIT192: DIFFERENTIAL CALCULUS FOR COMPUTER SCIENCE

DATE: 16/6/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

*Answer Question **ONE** which is compulsory and any other **TWO** Questions*

QUESTION ONE (30 MARKS)

a) Determine the $\lim_{x\to\infty}\frac{5x^2-3x+2}{10x^2-x+100}$ (4 marks)

b) State the $L'H\hat{o}pital's\ \ Rule$, hence or otherwise evaluate

$$\lim_{x\to -3}\frac{x+3}{x^2+5x+6}$$

(3 marks)

c) Determine $\frac{dy}{dx}$ given that $y=\frac{\sin x}{x}$ (3 marks)

d) Find the derivative of the following function from the first principles.

$f(x)=x^2$ (4 marks)

e) If $y=\frac{u^2-1}{u^2+1}$ and $u=\sqrt[3]{x^2+2}$ find $\frac{dy}{dx}$, (5 marks)

f) Given that $f(0)=8,\ g(0)=5,\ f'(0)=3, g'(0)=1$, Find $F'(0)$ where $F(x)=\frac{f(x)}{g(x)}+3x^2+4$

(3 marks)

g) Determine if the following function is continuous. $g(x)=\begin{cases}+1 if x \geq 0 \\ -1 if x < 0\end{cases}$ (4 marks)

h) Find the derivative the following functions $y = x^2 - 4x$ and $x = \sqrt{2t^2 + 1}$, at $t = \sqrt{2}$ using chain rule , (4 marks)

QUESTION TWO (20 MARKS)

a) Using logarithmic differentiation and differentiate the following functions;

$xe^x \sin x$ (3 marks)

$te^t \cos t$

b) Find the gradient of the curve $x = \frac{t}{1+t}$, and $y = \frac{t^3}{1+t}$ at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ (4 marks)

c) Determine $\frac{dy}{dx}$ given that $x^2 + 2xy - 2y^2 + x = 2$, at the point $(-4,1)$ (4 marks)

d) Write down the derivative of $\ln\left(\frac{1}{\sqrt{x}}\right)$ (4 marks)

e) Differentiate $y = 3x^2 + \cos 2x$ from the first principles. (5 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$ (3 marks)

b) Obtain $\frac{df(x)}{dx}$ for $f(x) = \frac{\sin x + e^{2x}}{\sin x}$ (4 marks)

c) Ink is dropped onto a blotting paper forming a circular stain which increases at the rate of $5cm^2/s$. Find the rate of change of the radius when the area is $30cm^2$ (6 marks)

d) Evaluate the derivative of the function given as $x^{\cos x}$ with respect to x, with respect to x, (7 marks)

QUESTION FOUR (20 MARKS)

a) If $x = t^3, y = t^2 - t$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ (6 marks)

b) Differentiate $y = \left(x^2+1\right)^3\left(x^3+1\right)^2$ (5 marks)

c) The curve of the function $f(x) = \alpha x^5 + \beta x^4 + 5x^3 - 1$ passes through (1,0) and has a stationary point at (1,0). Find the value of α , β , the other turning points and hence sketch the curve (9 marks)

QUESTION FIVE (20 MARKS)

a) Find $\frac{dy}{dx}$ given that $x^2 + 2xy - 2y^2 + x = 2$ at the point $(-4,1)$ (6 marks)

b) Determine the dimensions that would minimize the total surface area of an open rectangular container if it is to have a volume of $32m^3$ (8 marks)

c) Determine the domain for each of the following functions

i. $y = x^3 + 3x - 6$ (1 mark)

ii. $y = \frac{1}{x^2+6x+9}$ (2 marks)

d) Differentiate $x = t^3 + t^2, y = t^2 + t$ (3 marks)