MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 335: ORDINARY DIFFERENTIAL EQUATIONS I

SAC 300: DIFFERENTIAL EQUATIONS

DATE: **TIME:**

INSTRUCTIONS: Answer **question one** and **any other two** questions

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Given the differential equation $5\frac{d^2y}{dx^2} - 8x\frac{dy}{dx} + 2y = xy$, determine its order and degree (2 marks)

b) Solve the equation $(D^2 - 4D + 13)y = 0$ (4 marks)

c) Let A $= \begin{pmatrix} 12 & -15 \\ 4 & -4 \end{pmatrix}$, Determine the Eigen values of A. (4 marks)

d) Determine the differential equation corresponding to $y = e^{3x}(Acos3x + Bsin3x)$ where A and B are arbitrary constants. (5 marks)

e) Determine the particular solution of the differential equation $(yx^2 + y)dy - (xy^2 + x) = 0$, given that $x = 1, y = 1$ (5 marks)

f) Show that the differential equation below is exact and hence solve it. (5 marks)

$$(3x^2 + 4xy)dx + (2x^2 + 2y)dy = 0$$

g) Solve the initial value problem $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 0; y(0) = 2, y'(0) = -3.$ (5 marks)

QUESTION TWO (20MARKS)

a) Solve the equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 2x = 85sin3t$,given that when t=0, x=0 and $\frac{dx}{dt} = -20$. Show that the values of t for stationary values of the steady-state solution are the roots of 6tan3t=7. (10 marks)

b) A mass suspended from a spring performs vertical oscillations and the displacement x(cm) of the mass at time t(s) is given by $\frac{1}{2}\frac{d^2x}{dt^2} = -48x$. If $x = \frac{1}{6}$ and $\frac{dy}{dx} = 0$ when t=0, determine the period and amplitude of the oscillations. (10 marks)

QUESTION THREE (20MARKS)

a) Consider the differential equation $(4x + 3y^2)dx + 2xydy = 0$
   i) Show that this equation is not exact. (2 marks)
   ii) Find an integrating factor. (3 marks)
   iii) Using your integrating factor found in (ii) above solve the equation. (5 marks)

b) Solve the Bernoulli's equation below by using the transformation$u = y^{1-n}$. (10 marks)

$$\frac{dy}{dx} + x^5y = x^5y^7$$

QUESTION FOUR (20MARKS)

a) A cup of tea is prepared in a preheated cup with hot water so that the temperature of both the cup and the brewing tea is initially 190^0F. The cup is then left to cool in a room kept at a constant 72^0F. Two minutes later, the temperature of the tea is 150^0F. Determine;
   i) The temperature of the tea after 5 minutes. (4 marks)
   ii) The time required for the tea to reach 100^0F. (3 marks)

b) Solve the differential equation $cosy + (1 + e^{-x})siny\frac{dy}{dx} = 0$, given that $y = \frac{\pi}{4}, x = 0$ (5 marks)

c) Solve the equation $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = e^{-3t}$, given that $t = 0, x = \frac{1}{2}$and $\frac{dy}{dx} = -2$. (8 marks)

QUESTION FIVE (20MARKS)

a) Solve the equation $(D + 2)(D + 3)(D - 4)y = e^{5x}$ (6 marks)

b) Solve the Cauchy-Euler equation below by using the transformation $x = e^z$ (6 marks)

$$x^3\frac{d^3y}{dx^3} + x\frac{dy}{dx} - y = xlnx$$

d) Show that the equation $(x^2\_3y^2)dx + 2xydy = 0$ is a first order homogenous equation hence solve it by using the transformation $y = vx$. (8 marks)