



MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 462: OPERATIONS RESEARCH III

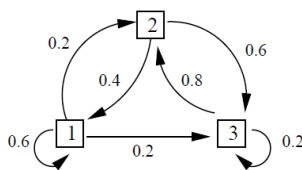
DATE:

TIME:

INSTRUCTION: Answer Question **ONE** which is compulsory and any other **TWO** Questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Distinguish the following terms as they apply in operations research
- i. Producer's surplus and Pareto optimality
 - ii. Kuhn Tucker analysis and cost-benefit analysis (4 marks)
- b) Explain the necessity of goal programming (3 marks)
- c) Distinguish the following Markov chain properties;
- i. Irreducible **and** Absorbing state
 - ii. Recurrent **and** Transient (4 marks)
- d) The figure given below shows the chances of a marketer of ABC Company being in any of the three cities in a week.



- i. Write the transition matrix, M to represent the movements.
 - ii. Using the matrix, determine $P(X_2 = 3 | X_0 = 1)$ (4 marks)
- e) Beta ltd co. has developed the following nonlinear programming model to determine the optimal number of Jars (x_1) and mugs (x_2) to produce per day. Maximize

$$Z = 7x_1 - 0.3x_1^2 + 8x_2 - 0.4x_2^2$$

subject to

$$4x_1 + 5x_2 = 100$$

Then determine the optimal solutions to the nonlinear programming model using the substitution method (8 marks)

- f) Kasiwa Ltd. produces two soft drinks Alpha and Berry. Both drinks are made of two food stuffs I and II. Production of one Kg of Alpha requires 7 units of food stuff I and 4 units of food stuff II whereas for producing one Kg of Berry, 4 units of food stuff I and 3 units of food stuff II are required. The company has 145 units of food stuff I and 90 units of food stuff II. The profit per Kg of Alpha is Ksh 120 while that of Berry is Ksh 90. The manager wants to earn a maximum profit of Ksh. 2700 and to fulfil the demand of 12 Kgs of Alpha. Formulate a goal programming problem for this situation. (7 marks)

QUESTION TWO (20 MARKS)

- a) Use Lagrangian multiplier method to solve the NLPP:

$$\text{Minimize } Z = x_1^2 + x_2^2 + x_3^2$$

$$\text{subject to } 4x_1 + x_2^2 + 2x_3 = 14$$

$$x_1, x_2 \geq 0$$

(10 marks)

- b) Solve using duality

$$\text{Minimize } Z = 0.7x_1 + 0.5x_2$$

$$x_1 \leq 4$$

$$\text{Subect to } x_2 \geq 6$$

$$x_1 + x_2 \geq 20$$

$$2x_1 + x_2 \geq 18$$

$$x_1, x_2 \geq 0$$

(10 marks)

QUESTION THREE (20 MARKS)

- a) The following inter-industry transactions table shows a two sector closed economy.

Inter-industry Transactions				
Industries	Input to agriculture	Input to manufacturing	Final demand	Total output
Agriculture	10	25	15	50
Manufacturing	20	30	10	60

Determine the following using the values in the table;

- The viability of the system as per Hawkins- Simon conditions (6 marks)
 - The outputs when the final demand changes to 35 and 42 for Agriculture and manufacturing respectively. (4 marks)
- b) The Lane industries manufacture pencils and rubbers. A pencil requires 1 hour of labor and 9 grams of wood. A rubber requires 1 hour of labor and 5 grams of wood to extract the latex. Currently 6 hours of labor and 45 units of wood are available. Each pencil contributes Ksh 8.00 to profit, and each rubber contributes Ksh 5.00 to profit. Formulate and solve an Integer Program (IP) to maximize Lane's profit. (10 marks)

QUESTION FOUR (20 MARKS)

- a) Discuss two factors that affects manpower planning in Kenya (4 marks)
- b) A company manufactures two products, radios and transistors, which must be processed through assembly and finishing departments. Assembly has 90 hours available, finishing can handle up to 72 hours of work. Manufacturing one radio requires 6 hours in assembly and 3 hours in finishing. Each transistor requires 3 hours in assembly and 6 hours in finishing. If profit is \$US 120 per radio and \$US 90 per transistor. Determine the best combinations of radios and transistors to realize a profit of \$US.1500 and to meet the demand of 10 radios, obtain the optimal solution (16 marks)

QUESTION FIVE (20 MARKS)

- a) The CEO of Beacons Real Estate company is indifferent which option to pursue between two options whereby; option 1 he would build 300 houses, renting 50 of them for 10 years at Ksh 300,000 per year. The 50 rented units would be sold after 10 years for Ksh 6,000,000. Construction costs for 300 houses would be Ksh 8,000,000 per house, which would sell for Ksh 10,000,000 each. The cost of a sales office would be Ksh 100,000,000 and the salaries of sales staff would be Ksh 20,000,000 each year. The project would last 2 years, with a financing cost of Ksh 200,000,000 per year. For option 2, the construction company could build 200 houses, renting 25 of them for 5 years at Ksh 300,500 per year. The 25 units could be sold after 5 years for Ksh 7,000,000. Construction costs for option 2 would be Ksh 7,000,000 per house, and the rest of the homes would sell for Ksh 11,000,000 each. The cost of a sales office would be Ksh 200,000,000 and sales staff salaries would be Ksh 15,000,000 each year. The project would last 1 year, with a financing cost of Ksh 100,500,000 per year. Based on the cost benefit analysis advice the CEO. (8 marks)
- b) Some historical stock exchange data shows that, for a week characterized by a bull market trend there is a 90% chance that another bullish week will follow. Additionally, there is a 7.5% chance that the bull week instead will be followed by a bearish one, or a 2.5% chance that it will be a stagnant. After a bearish week there's an 80% chance that the upcoming week also will be bearish, or 5% stagnant. The following week after the stagnant one there is 25% and 50% chance of bearish and stagnant respectively.
- i) Compile these probabilities into a transition matrix M: (6 marks)
- ii) By setting the current week as bearish determine the probabilities of a bull, bear, or stagnant in;
- i. One week's time from now (3 marks)
- ii. Fifty weeks' time from now (3 marks)