MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE
BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING
BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE
BACHELOR OF ECONOMICS AND STATISTICS
BACHELOR OF SCIENCE IN MATHEMATICS
BACHELOR OF EDUCATION
BACHELOR OF ARTS

SMA 260: PROBABILITY AND STATISTICS I

DATE: 14TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:

1. *Answer **Question** **1** and any other **two** questions.*
2. *You need a Scientific Calculator and Statistical Tables for this paper.*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

(a) (i) State *two* properties of a probability mass function (*p.m.f.*). (2 *marks*)

(ii) Explain any *two* conditions which must be satisfied by a binomial probability distribution. (4 *marks*)

(b) A continuous random variable x has a probability density function (*p.d.f.*) given by:

$$f(x) = \begin{cases} \frac{1}{40}(2x+3) \ ; & \text{for} \quad 0 \leq x \leq 5. \\ 0 \quad ; \quad \text{elsewhere} \end{cases}$$

Determine each of the following statistical measures about the random variable x.

(i) mean of x.

(ii) median of x. (6 *marks*)

(c) A continuous random variable x is given by the probability function:

$$f(x) = \begin{cases} k\,(6x - x^2 - 5) & \text{for} \quad 1 \leq x \leq 5 \quad \text{where} \quad k \text{ is a constant} \\ 0 \;\; ; \;\; \text{elsewhere} \end{cases}$$

If the function $f(x)$ is a probability density function (*p.d.f.*), then determine the value of the constant k. (5 *marks*)

(d) Research finding shows that in primary schools, the average number of left-handed learners is 5 students per school. Assuming that this finding is true, and assuming the Poisson distribution, determine the probability that in a school selected at random, the following number of learners will be left-handed:

(i) between 7 and 9 learners inclusive.

(ii) at least 3 learners. (6 *marks*)

(e) A discrete random variable X has a probability distribution given by:

X	0	1	2	3	4	5
$f(x)$	$\frac{2}{20}$	$\frac{3}{20}$	a	$\frac{5}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

Where, a is a numeric constant

(i) If $f(x)$ is a probability mass function, determine the value of the constant a;

(ii) Hence, determine the *mean* and the *variance* of the variable x. (7 marks)

QUESTION TWO (20 MARKS)

(a) State *two* properties of the normal probability distribution. (2 *marks*)

(b) A continuous random variable x has an exponential probability distribution with probability density function (*p.d.f.*) given by:

$$f(x) = \begin{cases} \lambda\, e^{-\lambda x} \;\; ; & \text{for} \quad 0 \leq x \leq \infty \quad \text{and} \quad \lambda > 0. \\ 0 \;\; ; & \text{elsewhere} \end{cases}$$

Show that the variance of x is given by: $var(x) = \frac{1}{\lambda^2}$ (8 *marks*)

(c) An ecologist claims that the lifespan of a tiger in a certain ecosystem is normally distributed with a mean age of 9.6 years and a standard deviation of 3.2 years. Assuming that this claim is true, do the following:

(i) Determine the proportion of tigers in the ecosystem whose age is between 7.6 years and 13.2 years. (4 *marks*)

(ii) If these tigers start hunting for prey at the age of 9 years, determine the proportion of these tigers that are capable of hunting prey. (3 *marks*)

(iii) Research findings show that the youngest 21% of these tigers are unable to hunt for prey. Based on these findings, determine the minimum age at which a tiger begins to hunt for prey. (3 *marks*)

QUESTION THREE (20 MARKS)

(a) A discrete random variable x has a discrete uniform probability distribution with probability mass function (*p.m.f.*) given by:

$$f(x) = \begin{cases} \frac{1}{n} & \text{for} \quad x = 0, 1, 2, 3, \ldots\ldots, n. \\ 0 \quad ; & \text{elsewhere} \end{cases}$$

Show that the variance of the random variable x is given by:

$$var(x) = \frac{1}{12}(n^2 - 1)$$ (6 *marks*)

(b) A discrete random variable x has a Poisson probability distribution with probability mass function (*p.m.f.*) given by:

$$f(x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!} & \text{for} \quad x = 0, 1, 2, 3, \ldots\ldots \\ 0 \quad ; & \text{elsewhere} \end{cases}$$

(i) Derive the moment generating function of x. (4 *marks*)

(ii) Hence, using the first and the second moments, determine the *mean* and the *variance* of the variable x. (6 *marks*)

(c) The age of staff members in a certain university has been found to be normally distributed such that 31% of the staff members are below 45 years of age, while 8% of the staff members are above 64 years of age. Determine the mean and the standard deviation of the age distribution of the university staff members. (4 *marks*)

QUESTION FOUR (20 MARKS)

A continuous random variable x has a probability distribution function given by:

$$f(x) = \begin{cases} \frac{3}{32}(8x - x^2 - 12) \ ; & \text{for} \quad 2 \leq x \leq 6. \\ 0 \ ; & \text{elsewhere} \end{cases}$$

Determine each of the following measures about the random variable x in the distribution:

(i) mode of x ; (3 *marks*)

(ii) mean of x ; (5 *marks*)

(iii) median of x ; (6 *marks*)

(iv) variance of x . (6 *marks*)

QUESTION FIVE (20 MARKS)

(a) Mr Kay has established that his supper takes between 80 to 120 minutes after placing an order in one of the hotels in Machakos town. Assuming a uniform continuous probability distribution, determine each of the following:

(i) The mean time of waiting;

(ii) The variance of the waiting time;

(iii) The probability that he waits for more than 95 minutes;

(iv) The probability that his waiting time is within one standard deviation from the mean.

(10 *marks*)

(b) A continuous random variable x has a probability density function given by:

$$f(x) = \begin{cases} 2x \ ; & \text{for} \quad 0 \leq x \leq 1. \\ 0 \ ; & \text{elsewhere} \end{cases}$$

(i) A continuous random variable y is defined by the relationship, $y = 8x^3$. Using the change of variable technique, determine the probability density function of y using the change of variable technique. (5 *marks*)

(ii) Hence, determine each of the following measures about the variable y:

I. the mean of y ;

II. the probability $P(Y > 4)$ (5 *marks*)