



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 302: GROUP THEORY

DATE: 22/7/2019

TIME: 11:00 – 1:00 PM

INSTRUCTIONS TO CANDIDATES

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A: (COMPULSORY)

QUESTION ONE (30 MARKS)

- a) Define the following terms
- i) Binary operation
 - ii) Transposition
 - iii) Permutation
 - iv) Set
 - v) Function (5 marks)
- b) Construct the multiplication table for A_3 . (5 marks)
- c) Construct the addition table for Z_{12} and show that $\{0,5\}$ and $\{0,1\}$ are not the proper subgroups and construct its lattice diagram. (6 marks)
- d) Every cyclic group is abelian. Proof (5 marks)

- e) If G is a group with binary operation $*$ and if a and b are any elements of G then the linear equation $y * a = b$ and $x * a = b$ have a unique solution in G . proof (5 marks)
- f) If $f: G \rightarrow G'$ is an isomorphism of G with G' and e is the identity of G then $f(e)$ is the Identity in G' . (4 marks)

SECTION B: ANSWER ANY OTHER TWO QUESTIONS

QUESTION TWO 20 MARKS

- a) Define a group (5 marks)
- b) Let A be a non empty set and let S_A be the collection permutations of A . Then S_A is a group under permutation multiplication. Proof (7 marks)
- c) Construct addition and the multiplication table Z_{16} , find its subgroups and draw its lattice diagram. (8 marks)

QUESTION THREE 20 MARKS

- a) Construct the multiplication table for D_4 (8 marks)
- b) The collection of all even permutations of a finite set of n elements form a subgroup of order $\frac{n!}{2}$ proof. (6 marks)
- c) Let G be a group and let $a \in G$. The $H = \{a^n | n \in \mathbb{Z}\}$ is a subgroup of G and is the smallest subgroup which contains a . proof (6 marks)

QUESTION FOUR 20 MARKS

- a) If G is a group with binary operation $*$ then the right and the left cancellation hold in G . proof (6 marks)
- b) Define a function $f: G \rightarrow G^1$ by $f(x) = axa^{-1}$ show that f is isomorphism. (6 marks)
- c) Construct the multiplication table for Klein 4-group and is $\{e, a, b\}$, $\{e, a, c\}$ and $\{e, b, c\}$ a subgroup of Klein 4-group. Construct the lattice diagram. (8 marks)

QUESTION FIVE 20 MARKS

- a) Let G be a group and H a non empty subset of G . Then H is a subgroup of G if and only if for $a, b \in H, ab^{-1} \in H$. prove (6 marks)
- b) In a group G with the operation $*$, there is only one identity and inverse (6 marks)
- c) State whether the following permutation is odd or even
- i) $(13245)(679)(1011)$ (4 marks)
- ii) $(134)(587)(26)$ (4 marks)