MACHAKOS UNIVERSITY

University Supplementary Examinations

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR BACHELOR OF EDUCATION AND BACHELOR OF ARTS , BACHELOR OF SCIENCE(MATHEMATICS), BACHELOR OF SCIENCE (MATHEMATICS & COMPUTER SCIENCE) AND BACHELOR OF ARTS.

SMA 305: COMPLEX ANALYSIS I

DATE:………………………………………………………….. TIME:…………………….

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE 30 Marks (Compulsory)

a. Find the fixed points of the bilinear transformation $w = \frac{z-1}{z+1}$ (3 marks)

b. If $w = f(z) = 3z + 4 - 5i$, find the value of w which corresponds to $z = -2 + i$ (3 marks)

c. Find the image of the point $4 + 5i$ on $z-$plane under the transformation $w = z + 1$ (4 marks)

d. Evaluate $\oint \frac{dz}{z-a}$ over a closed curve C and a is inside C (5 marks)

e. Find the roots of $z^3 = 8$ (5 marks)

f. Find the value of the derivative of $f = (z - 4i)^8$ at $z = 3 + 4i$ (5 marks)

g. Find the residue of $f(z) = \frac{z^2}{(z-1)(z+2)}$ at all its poles (5 marks)

QUESTION TWO (20 MARKS)

a. Given the function $f(z) = \frac{e^{3z}}{(z-2)^3}$

i. State the singularity of $f(z)$ (2 marks)

ii. What is the kind of singularity of $f(z)$ (2 marks)

iii. Expand $f(z)$ in a Laurent series (5 marks)

iv. State the region of convergence of $f(z)$ (1 mark)

b. Verify that $u = x^2 - y^2 - y$ is harmonic in the whole complex plane. Hence find harmonic conjugate function v of u. Determine the analytic function $f(z)$ (10 marks)

QUESTION THREE (20 MARKS)

a. Find the values of z such that $e^z = 2$ (5 marks)

b. Expand $f(z) = \frac{1}{(z-1)(z-3)}$ about $z = 3$ in Laurent series (5 marks)

c. Given the complex function $w = f(z) = u(x, y) + iv(x, y)$ derive the Cauchy Riemann's equations in both rectangular coordinates and in polar form (10 marks)

QUESTION FOUR (20 MARKS)

a. Determine whether $f(z) = \frac{3z^4-2z^3+8z^2-2z+5}{z-i}$ is continuous at $z = i$. Hence determine its limit at $z = i$ (5 marks)

b. Prove that $\cos z = cosxcoshy - isinxsinhy$ (5 marks)

c. Find the bilinear transformation which maps the points $z = 0, -i, -1$ on the Z-plane onto the points $w = i, 1, 0$ respectively on the w-plane. (10 marks)

QUESTION FIVE (20 MARKS)

a. Determine the poles of $\frac{e^{iz}}{z^2(z^2+2z+2)}$ (5 marks)

b. Show that $f(z) = e^z$ is a periodic function with period $2k\pi i$ (5 marks)

c. Show that $\int_{3+4i}^{4-3i} (6z^2 + 8iz)dz$ has the same value the following paths; C joining the

Points $3+4i$ and $4-3i$ along the straight line and also along the straight line $3+4i$ to $4+4i$, then from $4+4i$ to $4-3i$ (10 marks)