MACHAKOS UNIVERSITY

University Examinations for 2023/2024 Academic Year

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

FIFTH YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

EMM 514: CONTROL ENGINEERING II

DATE: **TIME:**

INSTRUCTIONS

- This paper consists of Five Questions.
- **Question 1** is **Compulsory.** Attempt **Any Other Two** Questions.
- Answers to any particular question must be together.

QUESTION ONE (COMPULSORY) (30MARKS)

Figure Q 1 shows mass-spring-damper system. The states can be defined as

$$x_1 = y$$

$$x_2 = \frac{dy}{dt} = \dot{x_1}$$

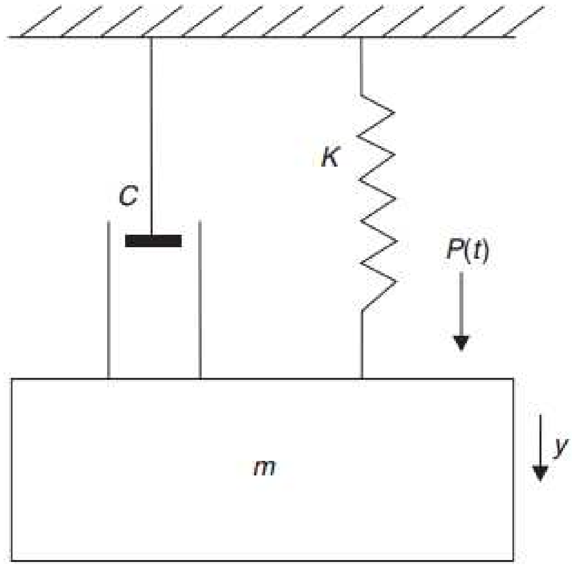

Figure Q 1

a) Show that (5marks)

$$\dot{x(t)} = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Where

$$A = \begin{bmatrix} 0 & 1 \\ -k/_m & -c/_m \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/_m \end{bmatrix}, C = [1 \quad 0] \text{ and } D = 0$$

b) Given that $m = 1kg, C = 3Ns/m, K = 2N/m$ and $u(t) = 0$. Determine the following

i) Characteristic Equation, Un-damped natural frequency and damping ratio (2 marks)

ii) Transition matrixes, $\phi(s)$ and $\phi(t)$ (4 marks)

iii) Transient response of the state variables to a unit step input (3 marks)

c) Assuming zero initial conditions that is, $\dot{y}(0) = 1.0$ and $y(0) = 0$; evaluate the transient response for the system in b) above due to a unit step input using

i) Convolution Integral (4 marks)

ii) Inverse Laplace Transform (4 marks)

d) For the system in b) above, determine the *discrete-time transition and control matrices* using a sampling time $T = 0.1s$ (4 marks)

e) Using *Matrix Vector Difference Equation Method (MVDEM),* determine the transient response of the state variables in b) above due to a unit step input (4 marks)

QUESTION TWO (20MARKS)

a) The state representation for a control system is defined by the relation

$$\dot{x(t)} = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

State the condition for complete observability (2 marks)

A control system is defined by the equation

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \ \gamma = 0.5, \omega_n = 10rad/s$$

b) Find the eigen values for the following cases

i) Open Loop (2 marks)

ii) Closed Loop (3 marks)

c) Design a Full Order Observer using the following approaches

i) Direct Comparison Method (DCM) (5 marks)

ii) Observer Canonical Form (OCF) (4 marks)

iii) Ackermann's Formula (AF) (4 marks)

QUESTION THREE (20MARKS)

a) Beginning from a well labelled Circuit Diagram, derive the closed loop gain of a PI Controller (3 marks)

b) A liquid level process control system is as shown in Figure Q 3a) while the Block Diagram for the same is as shown in Figure Q 3b)

i) Compute the closed loop transfer function for the system (4 marks)

ii) Derive the expressions of the natural frequency and damping ratio of the system (3 marks)

c) The system parameters are for the system described in 3b) above are $A = 2m^2, R_f = 15s/m^2$, $H_1 = 1V/m$, $K_v = 0.1m^3/sV$, $K_1 = 1$ (controller gain) and the undamped natural frequency is 0.1 rad/s.

i) Determine the Integral Action Time and Damping Ratio of the System (3 marks)

ii) Assuming zero initial conditions, find an expression for the time response of the system when there is a step change of $h_d(t)$ from 0 to 4m. Sketch such a response (7 marks)

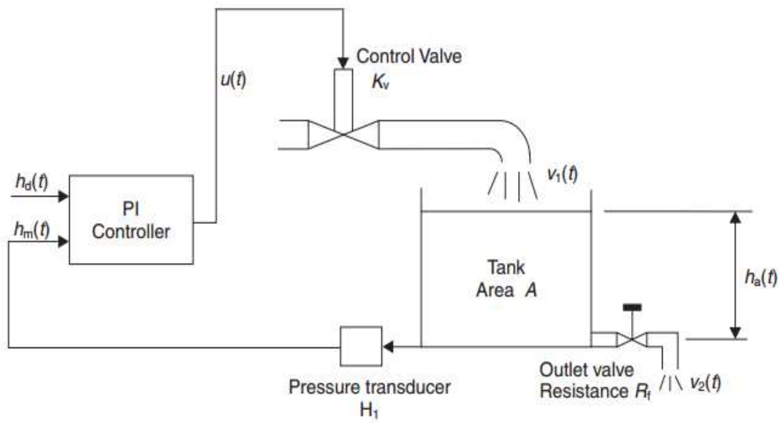

Figure Q 3a)

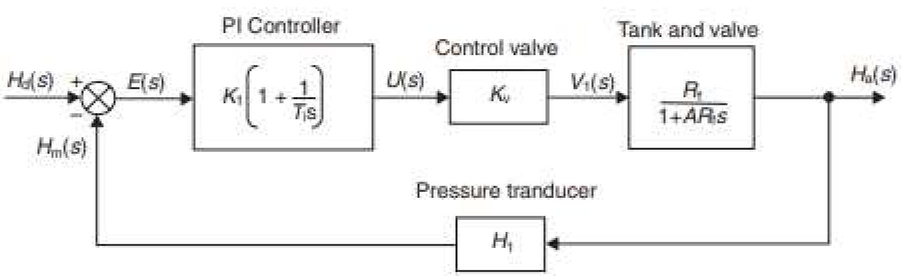

Figure Q 3b)

QUESTION FOUR (20MARKS)

a) The state representation is defined by the relation

$$\dot{x(t)} = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Show that

$$x(t) = \phi(t)X(o) + \int_0^t \phi(t-\tau)\, Bu(\tau)d\tau$$

$$y(t) = C\phi(t)X(o) + \int_0^t C\phi(t-\tau)\, Bu(\tau)d\tau + Du(t$$

All the symbols carry their usual meaning (6 marks)

b) For a Control Systems defined by the transfer functions, determine the following

$$G(s) = \frac{Y(s)}{R(s)} = \frac{1}{S^2 + 3S + 2}$$

i) Matrix differential equations for the system in OCF (2marks)

ii) State transition matrix $\phi(t)$ (5 marks)

iii) Zero-state response if a unit step is applied. Use the Convolution Integral for the initial condition, $X(o) = [1,0]$ (7 marks)

Question 5 (20marks)

a) Define Proportional (P) Controller (2marks)

b) A cruise controller for a car that uses a simple Proportional (P) controller is to be designed. The controller is no more than a simple amplifier ($C(s) = K_p$). Consider a car of mass 1,000kg, a friction constant of 50Ns/m and an output force of the engine being 500N. Assume zero initial conditions for the output. Design the Control System by:

   i) Modelling the car dynamics (3marks)

   ii) Obtaining the *Open Loop* response (3marks)

   iii) Obtain the *Closed Loop* response (7marks)

   iv) Determine the closed loop response with *disturbance* (for example wind) (5marks)