



MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS

FOURTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE IN STATISTICS & PROGRAMMING

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF ARTS

SMA 460: STOCHASTIC PROCESSES

DATE:13/8/2021

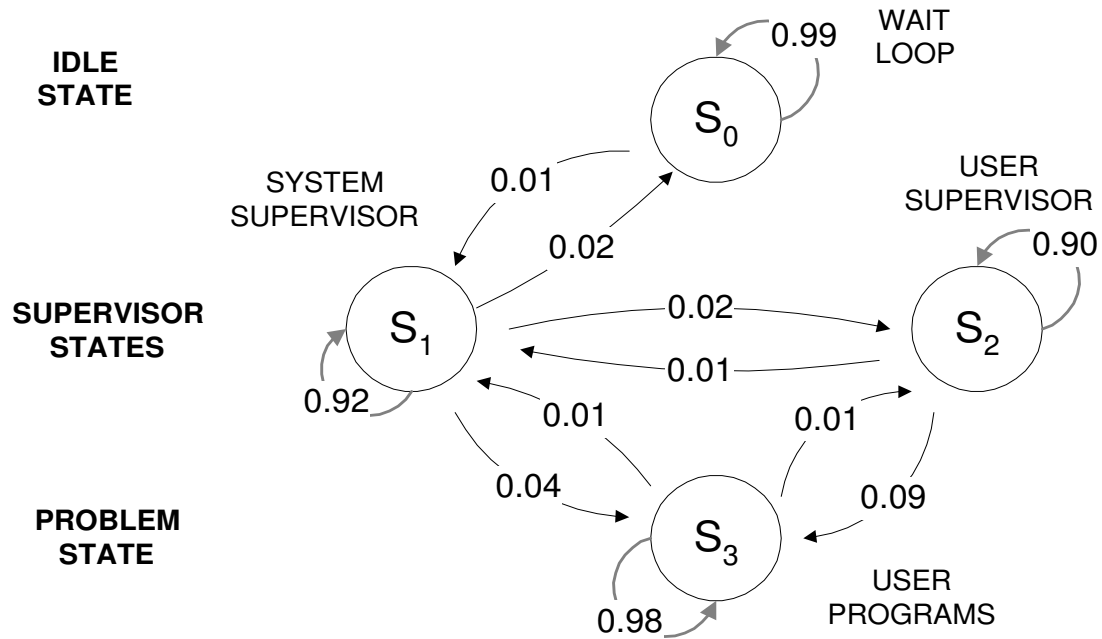
TIME: 2:00 – 4:00 PM

INSTRUCTIONS

Answer question ONE and any other TWO questions

QUESTION ONE (COMPULSORY) (30 MARKS)

- a) Describe the following terms as used in stochastic processes by giving examples
 - (i) Stochastic process (2 marks)
 - (ii) State (2 marks)
 - (iii) Transition (2 marks)
- b) Describe the main components that make up a queue? (3 marks)
- c) State Transition Diagram of discrete-time Markov of a CPU is as shown in the network diagram below: -



- (i) Draw and describe the state Transition Probability Matrix; (3 marks)
- (ii) Calculate the equilibrium state probabilities by solving system of equations. (3 marks)
- d) Describe the markovian property giving an example. (3 marks)
- e) Describe with examples the following markov chain properties:
 - (i) Accessible (2 marks)
 - (ii) Communicable (2 marks)
 - (iii) Irreducible (2 marks)
 - (iv) Transient (2 marks)
 - (v) Recurrent (2 marks)
 - (vi) Absorbing (2 marks)

QUESTION TWO (20 MARKS)

- a) An insurance company classifies drivers as low-risk if they are accident-free for one year. Past records indicate that 98% of the drivers in the low-risk category (L) will remain in that category the next year, and 78% of the drivers who are not in the low-risk category (L') for one year will be in the low-risk category the next year.
 - (i) Determine the transition matrix, P (5 marks)

- (ii) If 90% of the drivers in the community are in the low-risk category this year, determine the probability that a driver chosen at random from the community will be in the low-risk category
- the next year?
 - The year after next ?
- (5 marks)
- b) (i) State and describe the Chapman Kolmogorov Equations. (5 marks)
- (ii) Consider an IRS auditing example:

Two states: $s^0 = 0$ (no audit), $s^1 = 1$ (audit)

With Transition matrix

$$\mathbf{P} = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$$

Determine the probability that there is no audit 2 years from now given that in the current year, there was no audit is. (5 marks)

QUESTION THREE (20 MKS)

- a) The probabilities of weather conditions, given the weather on the preceding day, can be represented by a transition matrix:

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

The matrix P represents the weather model in which a sunny day is 90% likely to be followed by another sunny day, and a rainy day is 50% likely to be followed by another rainy day. The columns can be labelled "sunny" and "rainy" respectively, and the rows can be labelled in the same order. $(P)_{ij}$ is the probability that, if a given day is of type i , it will be followed by a day of type j .

- Give a prediction of the weather on day 1. (2 marks)
 - Give a prediction of the weather on day 2. (2 marks)
 - By solving the pair of simultaneous equations give the steady state distribution (3 marks)
- b) Consider a communication system which transmits the digits 0 and 1 through several stages. At each stage the probability that the same digit will be received by the next stage, as transmitted, is 0.75.
- Draw the transition probability matrix (2 marks)
 - What is the probability that a 0 that is entered at the first stage is received as a 0 by the 5th stage? (3 marks)

- c) A petrol station owner is considering the effect on his business (Superpet) of a new petrol station (Global) which has opened just down the road. Currently (of the total market shared between Superpet and Global) Superpet has 80% of the market and Global has 20%.

Analysis over the last week has indicated the following probabilities for customers switching the station they stop at each week:

		To	
From	Superpet	0.75	0.25
	Global	0.55	0.45

- What will be the expected market share for Superpet and Global after another two weeks have past? (4 marks)
- What would be the long-run prediction for the expected market share for Superpet and Global? (4 marks)

QUESTION FOUR (20 MARKS)

- a) (i) Consider a markov chain with probability matrix shown below: -

$$\mathbf{P} = \begin{bmatrix} 0.8 & 0 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0 & 0.9 & 0.1 \end{bmatrix}$$

- Draw its corresponding state network diagram (3 marks)
- Chain is ergodic. (2 marks)

- b) Two aging computers are used for word processing. When both are working in the morning, there is a 30% chance that one will fail by the evening and a 10% chance that both will fail. If only one computer is working in the morning, there is a 20% chance that it will fail by the evening. If neither is working in the morning, the office sends all work to a typing service. Computers that fail during the day are picked up the following morning, repaired, and then returned the next morning. The system is observed after the repaired computers have been returned and before any new failures occur.

- Describe for the above computer repair problem, the current states, events, assigned probabilities, and next states for the above markov chain. (5 marks)
- Draw the associated probability transition matrix for the computer repair problem (5 marks)
- Draw a State transition network diagram (5 marks)

QUESTION FIVE (20 MARKS)

- a) Differentiate between discrete and continuous random variables by giving an example for each. (4 marks)
- b) Describe the following discrete probability distributions (6 marks)
- (i) Binomial distribution
 - (ii) Geometric distribution
 - (iii) Poisson distribution
- c) Describe the following Continuous probability distributions (6 marks)
- (i) Exponential distribution
 - (ii) Gamma distribution
 - (iii) Normal distribution
- d) In analysing switching by Business Class customers between airlines, the following data has been obtained by British Airways (BA):

		Next flight by	
		BA	Competition
Last flight by	BA	0.85	0.15
	Competition	0.10	0.90

For example if the last flight by a Business Class customer was by BA, the probability that their next flight is by BA is 0.85. Business Class customers make 2 flights a year on average.

Currently BA have 30% of the Business Class market. Forecast the BA's share of the Business Class market after two years? (4 marks)