MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION

SMA 362: OPERATIONS RESEARCH I

DATE: 12/04/2023 **TIME: 08:30 – 10:30 PM**

Instructions to the Candidate:

1. *Answer* ***Question 1*** *and any other* ***two*** *questions.*
2. *You must have a Scientific Calculator and Graph Paper.*

QUESTION ONE (COMPULSORY) (30 MARKS)

(a) (i) Explain **two** parts of a linear programming model. (4 *marks*)

(ii) Outline any **four** sources of constraints in an optimisation problem involving linear programming. (2 *marks*)

(b) Explain any **two** of the following types of variables as used in linear programming, in the context of both an initial feasible solution and an optimum solution:

(i) Slack variable;

(ii) Surplus variable;

(iii) Artificial variable. (4 *marks*)

(c) Given the linear programming model below:

$$\begin{aligned} \text{Minimise : } \quad & z = 8x_1 + 12x_2 + 15x_3 \\ \text{Subject to : } \quad & 2x_1 + 3x_2 + 6x_3 \geq 20 \\ & 6x_1 + 8x_2 + 5x_3 \geq 40 \\ & 7x_1 + 2x_2 + 4x_3 \geq 50 \\ \text{With : } \quad & x_1,\ x_2,\ x_3 \geq 0 \end{aligned}$$

Determine its symmetrical dual program. (2 *marks*)

(d) A soft drink production plant produces two quantities of soda: 300 ml and 500 ml. The sale prices per bottle are Ksh 30 for 300 ml and Ksh 45 for 500 ml. The firm wants to determine the hourly production plan which maximises contribution to its sales revenue. The production constraints are as shown below:

	Machine hours	Labour hours	Colouring agent
300 ml	1	2	1
500 ml	2	1	1
Maximum available	120	140	80

A customer has placed an order that a minimum of 20 bottles of 300 ml must be produced per hour in conformity with their consumption.

(i) Formulate a linear programming model for the problem. (3 *marks*)

(ii) Using the *graphical method*, determine the optimum production plan which maximises the hourly revenue for the bottling firm. (6 *marks*)

(iii) Interpret the optimum solution obtained in (ii) above. (3 *marks*)

(e) A shipping company is required to meet the demands at various destinations by transporting 80, 100, 40, 120, 60 containers respectively from various supply sources with 120, 80, 160, 40 containers. The transport cost in thousand Kenya shillings per unit of container over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 4 & 8 & 2 & 5 & 9 \\ 3 & 6 & 1 & 4 & 2 \\ 6 & 2 & 8 & 3 & 5 \\ 5 & 1 & 2 & 4 & 3 \end{bmatrix}$$

Sources: (120, 80, 160, 40)

Destinations: (80, 100, 40, 120, 60)

The company wants to meet the demand at the destinations by transporting the containers at the cheapest cost possible. Derive the *initial basic solution* and its corresponding total cost of transport using each of the following methods:

(i) Northwest corner (NWC) rule;

(ii) Vogel's approximation method (VAM). (6 *marks*)

QUESTION TWO (20 MARKS)

A fruit farm which produces three types of fruit juices – mango, orange and pineapple, has its daily production plan for sales revenue modelled as an LP program as shown below.

Maximise :	$z = 80x_1 + 120x_2 + 150x_3$	
Subject to :	$2x_1 + 3x_2 + x_3 \leq 1500$	Machine hours
	$2x_1 + x_2 + 5x_3 \leq 1200$	Preservative
	$x_1 + 2x_2 + x_3 \leq 750$	Colouring agent
	$x_2 \leq 250$	Comesa trade agreement
With :	$x_1, \; x_2, \; x_3 \geq 0$	Non-negativity conditions

where x_1 - the number of packets of mango juice produced,
x_2 - the number of packets of orange juice produced, and
x_3 - the number of packets of pineapple juice produced.

(i) Using the Simplex method, determine the optimum weekly production plan for the farm. (16 *marks*)

(ii) Interpret the optimum solution obtained in (i) above. (4 *marks*)

QUESTION THREE (20 MARKS)

Below is a linear programming model for a given problem. Use it to answer the questions that follow.

Minimise :	$z = 180x_1 + 120x_2 + 80x_3$
Subject to :	$2x_1 + 2x_2 + x_3 \geq 80$
	$3x_1 + x_2 + x_3 \geq 70$
With :	$x_1, \; x_2, \; x_3 \geq 0$

(i) Determine the *symmetrical dual* of the linear programming model above. (2 *marks*)

(ii) Using the Simplex method, determine the optimum solution of the dual program for the linear programming model above. (10 *marks*)

(iii) Using the relationship between a primal program and its symmetrical dual program, extract the optimum solution of the primal program from the optimum solution of its dual program. (4 *marks*)

(iv) Interpret the optimum solution of the primal program for the linear programming problem. (4 *marks*)

QUESTION FOUR (20 MARKS)

An oil distribution company is required to meet the demands 70, 50, 40, 20 in thousand litres at various destinations from supplies 60, 20, 100 in thousand litres from various sources. The transport cost per unit amount (thousand Kenya shillings) over the various routes are as given in the matrix below:

$$C = \begin{bmatrix} 2 & 3 & 11 & 7 \\ 1 & 0 & 6 & 1 \\ 5 & 8 & 15 & 9 \end{bmatrix}$$

The company wants to meet the demand at destinations by transporting the oil at the cheapest cost possible. Using the northwest corner (NWC) rule, do the following for the transportation problem:

(i) Determine the optimal solution that minimizes the cost of transport; (17 *marks*)

(ii) Interpret the optimum solution obtained in (i) above. (3 *marks*)

QUESTION FIVE (20 MARKS)

(a) Outline *four* characteristics of an assignment problem. (4 *marks*)

(b) In solving a transportation problem using a transportation table, an initial table may be balanced or not. Explain the term *balanced* as used in this context. (3 *marks*)

(c) A computer networking company has its network technicians based at different locations in the county to maintain the networks at the premises of their various customers. Four maintenance requests have been received, and four technicians are available on standby. The distance in kilometres each of the technicians is away from the various customer premises is as shown in the table below.

		Customer			
		C1	C2	C3	C4
Technician	T1	19	23	16	25
	T2	15	21	20	22
	T3	22	18	17	21
	T4	14	24	16	22

The company intends to assign the technicians to the customers in such a way that the total distance to be travelled is minimised. Determine the optimal assignment of the technicians to the customer premises that will minimise the total distance travelled.

(13 *marks*)