MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

MASTER OF SCIENCE (MATHEMATICS)

SMA 810: FINITE ELEMENT METHOD

DATE: 27/1/2022 **TIME: 10:00 – 1:00 PM**

INSTRUCTIONS TO CANDIDATES

I. This paper consists of **FIVE** questions. Answer **QUESTION ONE** *(30 marks)* and **ANY OTHER TWO** *(20 marks each).*

II. Cheating will lead to automatic disqualification.

QUESTION ONE(COMPULSORY) (30 MARKS)

a) Consider the equation:

$$\frac{d^2u}{dx^2} + u + x = 0, \qquad 0 < x < 1$$

with $u(0) = u(1) = 0$. Assume an approximation for $u(x) = a_1\phi_1(x)$ with $\phi_1(x) = x(1-x)$. The residual is

$$R(u,x) = -2a_1 + a_1[x(1-x)] + x$$

and the weight $W_1(x) = \phi_1(x) = x(1-x)$. Find the solution from the following integral $\int_0^1 W(x)R(u,x)dx = 0$ (6 marks)

b) Figure 1 shows a system of three springs fastened at a wall. The three springs constitute finite elements of the assembly system.

(i) Assemble the stiffness matrix for the assemblage by superimposing the stiffness matrices of the springs. Here *k* is the stiffness of each spring. (6 marks)

(ii) Find the *x* and *y* components of deflection of node 1. (6 marks)

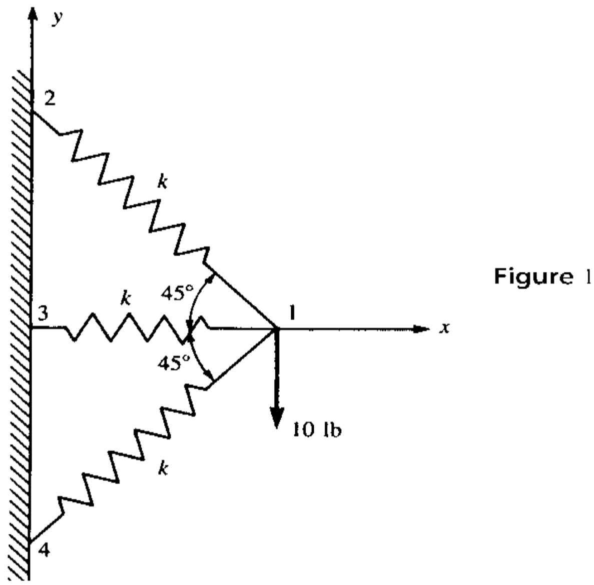

Figure 1

c) For the three element spring system shown in Figure 2 K_1= 100 N/mm, K_2 = 200 N/mm, K_3 = 100 N/mm, and P = 500 N.

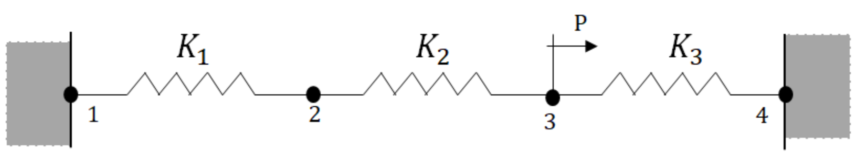

Figure 2

Determine

(i) The global stiffness matrix (3 marks)

(ii) Displacement at nodes 1 and 4. (3 marks)

(iii) Reaction forces at nodes 1 and 4. (3 marks)

(iv) The force in the spring 2. (3 marks)

QUESTION TWO (20 MARKS)

a) The deflection of a simply supported beam is governed by the following equation and boundary conditions:

$$E_1 \frac{d^2y}{dx^2} + M = 0 \text{ and } y(0) = y(L) = 0$$

where E_1 is a constant and M is a distributed bending moment. Derive the weak statement form of the governing equation. (8 marks)

b) A steel bar of 600 mm length has a cross-sectional area of $650\ \mathrm{mm}^2$ and $350\ \mathrm{mm}^2$ at two ends. It is fixed at the large end and subjected to two axial forces of 40 kN and 10 kN as shown in Figure 3. The modulus of elasticity E force the bar material is 200×10^3 $\mathrm{N/mm^2}$. Model the bar with three finite elements and determine

(i) Nodal displacements (4 marks)

(ii) Stress in each element (4 marks)

(iii) Reaction force at the support. (4 marks)

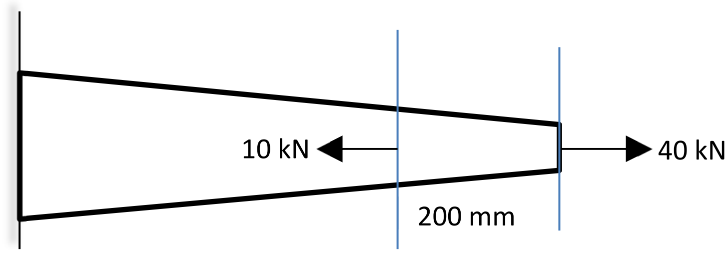

Figure 3

QUESTION THREE (20 MARKS)

a) For the plane trusses shown in Figures 2a and 2b, determine the horizontal and vertical displacements of node 1 and the stresses in each element. All elements have $\mathrm{E} = 210\ \mathrm{GPa}$ and $A = 4.0 \times 10^4\ \mathrm{m}^2$. (10 marks)

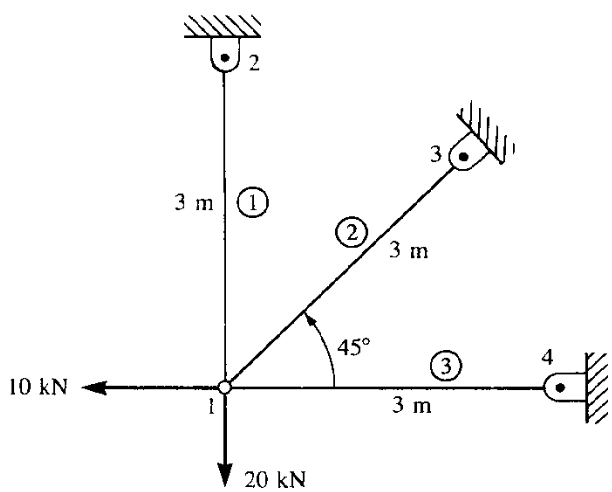

Figure 2 a

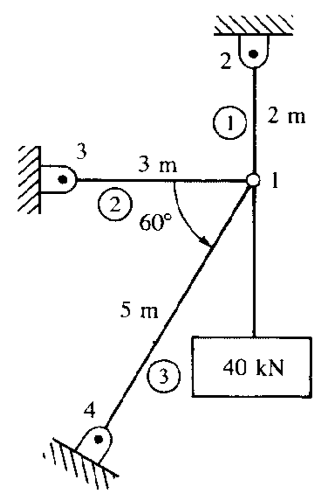

Figure 2 b

b) Remove element1 from Figure 2b and solve the problem. Compare the displacements and stresses to the results for Figure 2b in Question (a). (5 marks)

c) Use two 1-D bar elements to model the structure shown in Figure 4 which is loaded with force P, and constrained at two ends. Obtain the stress in the two elements.

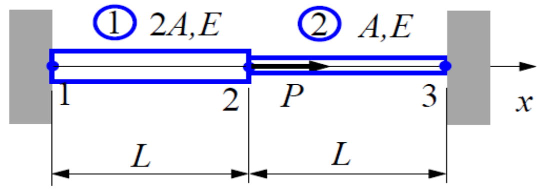

Figure 4

Note: stress $\sigma = E\epsilon. E$ is Young's modulus, $\epsilon = u/L$ is strain and u is displacement nodes.

(5 marks)

QUESTION FOUR (20 MARKS)

a) Consider the function $J[y] = \int_0^1 (x^2 - y^2 + \frac{dy}{dx})\, dx$

With $y(0) = 0$ and $y(1) = 1$. Calculate Δ J and δ J|y; η| when $y(x) = x$ and $\eta(x) = x^2$.

(6 marks)

b) Solve the differential equation

$$\frac{d^2y}{dx^2} + y = x, \qquad x \in [0,2]$$

with the boundary conditions $y(0) = 0, y(2) = 5$ using Galerkin method. (8 marks)

c) Consider a cantilever beam of variable flexural rigidity, $EI = a_0[2 - \left(\frac{x}{L}\right)^2]$ and carrying a distributed load, $q = q_0[1 - (x/L)]$. Find a three-parameter solution using the collocation method. (6 marks)

QUESTION FIVE (20 MARKS)

a) Calculate the stiffness matrix for the structure shown in Figure 5. The bar is vertical. (6 marks)

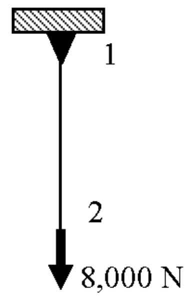

Figure 5

Element area $A = 1.5 \text{ m}^2$,

Young's modulus $\text{E} = 30{,}000{,}000$

Element length $L = 5 \text{ m}$

b) Using a different load for question (a) above, the element shown deforms by 0.02 m in length. What is the stress in the material? Use a finite element approach to solve this problem.

(6 marks)

c) For the spring assemblage shown in Figure 6, obtain the global stiffness matrix by the direct stiffness method. Let $k^{(1)} = 1$, $k^{(2)} = 2$, $k^{(3)} = 3$, $k^{(4)} = 4$, and $k^{(5)} = 5$. (8 marks)

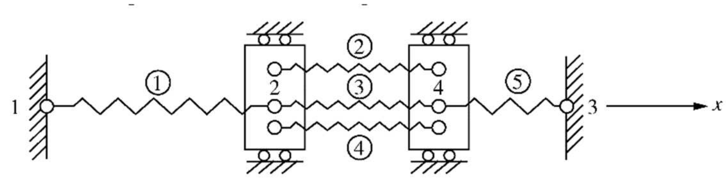

Figure 6