MACHAKOS UNIVERESITY

UNIVERSITY EXAMINATION, 2018/2019

EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SUPPLEMENTARY EXAMINATION

SMA 465: MEASURE AND PROBABILITY

Attempt Question 1 and any other 2 questions

Date: **Time:**

<u>QUESTION ONE</u> (30 marks).

a) Consider a non-empty set Ω. Define the following:

i. A measurable space,(Ω, Π)

ii. A measure on (Ω, Π), say , μ

iii. A measure μ whichis such that it is finite μ-a.e.

(12 marks).

b) State:

i. The Lebesgue Monotone Convergence Theorem.

ii. Fatou's Lemma (6 marks)

c) Prove any one of the results stated in Part (c). (4 marks)

d) Let P be a Probability measure. If **A** and **B** are some two events,

i. Define the conditional probability of**A** given **B**.

ii. Show that the conditional Probabilityof **A** given **B** is a Probability measure.

(8 marks)

QUESTION TWO(20marks).

a)

i. State the axioms of a measure, μ.

ii. Show that the function , m, defined on the theBorel algebra, B , by:

. m : $(\Re, \mathrm{B}) \rightarrow (\Re, \mathrm{B})$

.such thatm $\{(a_i,b_i]\} = b_i - a_i$, with a_i, b_i in $\Re$, $a_i \leq b_i$, for i=1,2,.... , is a measure.

. (10 marks).

b) Explain, for a given measure space(Ω, Π, μ),

i. Completion of the measure space

ii. μ -a.e

iii. the Lebesgue-Stieltjes measure space

. (10 marks).

QUESTION THREE. (20marks).

a) Show that for a non-empty set Ω:

i. The set $\{\Phi, \Omega\}$ is a σ-field

ii. The set $\{\Phi, A, A^c, \Omega\}$for $A \subset \Omega$,is aσ-field

(10marks).

b) Show that if Π is a σ-field then $\bigcap_{i=1} A_i \in \Pi$ for all $A_i \in \Pi$, i = 1, 2, 3,_

(10marks).

<u>QUESTION FOUR</u>(20marks).

a) Consider a non-empty set Ω. Define the following:

iv. A measurable space,(Ω, Π)
v. A measure on (Ω, Π), say , μ
vi. A measure μ whichis such that it is finite μ-a.e.

(12 marks).

b) Consider a function, **f**, which is given by:

$f : (\Omega_1, \Pi_1) \rightarrow (\Omega_2, \Pi_2)$

i. when is the function **f** said to be a measurable function? (4 marks)
ii. Suppose$\Omega_1 = \Omega_2 = \Re \cup \{-\infty, \infty\}$ and$\Pi_1 = \Pi_2$ = Borel opensubsetsin $\Re \cup \{-\infty, \infty\}$. Show that the constant function , for some c in $\Re$;f(x)=c, is a measurable function.

. (8 marks)

<u>QUESTION FIVE</u>. (20 marks)

a) State the Chebychev's inequality. (5marks).
b) Prove the Chebychev's inequality. (5 marks)
c) Let X_n: n ≥ 1 be a sequence of random variables with $E(X_n) = \boldsymbol{\mu}$,

for all X_n: n ≥ 1 . Show that the sample mean $\bar{X}_n$ converges to **μ** in probability.

. (10 marks).