



MACHAKOS UNIVERSITY

University Examination 2018/2019

SCHOOL OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

FIFTH YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

EEE 512: DIGITAL SIGNAL PROCESSING

DATE: 13/5/2019

TIME: 8.30-10.30 AM

INSTRUCTIONS

Answer Question ONE and any other TWO Questions.

QUESTION ONE (COMPULSORY-30 MARKS)

a) Given two sequences

$$x_1[n] = 3\delta[n] + 2\delta[n - 1]$$

$$x_2[n] = \delta[n - 1] + 2\delta[n]$$

i. Find the Z-transform of their convolution $Z\{x_1[n] + x_2[n]\}$

ii. Determine the convolution using Z-transforms (5 marks)

b) Find the inverse Z-transform of

$$H(z) = \frac{10z}{z^2 + z + 1} \quad (3$$

marks)

c) A discrete system is described by the difference equation $y[n] = 3x[n] + 5$. Determine whether the system is

i. linear

ii. time-invariant (6 marks)

- d) i Distinguish between decimation in time and decimation in frequency FFT algorithms
- ii Explain how pipelining improves the operation of a DSP processor. (4 marks)
- e) Given a sequence $x[n]$ for $0 \leq n \leq 3$ where $x(0) = 1, x(1) = 2, x(2) = 3, x(4) = 4$. Evaluate its DFT $X(k)$. (4 marks)
- f) A discrete-time processing operation is defined by the recurrence equation
- $$y[n] = x[n] + 2x[n - 1] + 3x[n - 3]$$
- Determine the unit sample response $h[n]$ of the processor (3 marks)
- g) Given a second order transfer function
- $$H(z) = \frac{0.5(1 - z^{-2})}{1 + 1.3z^{-1} + 0.36z^{-2}}$$
- Perform the filter realization and write the difference equations using the following realizations:
- i. Direct I form
- ii. Direct II form (5 marks)

QUESTION TWO (20 MARKS)

- a) A signal is to be digitally filtered using a digital filter based on an analogue second-order Butterworth filter with a normalized transfer function

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$$

The signal sampling frequency is 2KHz and the filter cut-off frequency is 600Hz.

- i. Using bilinear transformation, obtain the transfer function of the digital filter
- ii. Draw a realization of the filter structure using minimum number of delay elements (10 marks)
- b) The transfer function of an LTI system is
- $$H(z) = \frac{z}{(z - 0.75)(z + 0.5)}$$

- i. For each possible region of convergence, find the corresponding impulse response of the system
 - ii. State the ROC where the system is causal and the ROC for which it is stable
- (10 marks)

QUESTION THREE (20 MARKS)

- a) Distinguish between decimation-in-time and decimation in frequency as used in FFT
(2 marks)
- b)
 - i Starting from the from the definition of the DFT of $X[k] = \sum_{n=0}^{N-1} W_N^{nk}$, and using a 4-point DFT, show that the transform can be expressed as $\bar{X}[k] = Wx[n]$, where $\bar{X}[k]$ and $x[n]$ are column matrixes and W a transformation matrix.
 - ii Using the transformation Matrix in (i), determine the DFT of the sequence

$$x(n) = \{ \underset{\uparrow}{1}, 2, 2, 1 \}$$

(10 marks)
- c) By means DFT and IDFT, determine the response of the FIR filter with impulse response

$$h(n) = \{ \underset{\uparrow}{1}, 2, 3 \}$$

to the input sequence

$$x(n) = \{ \underset{\uparrow}{1}, 2, 2, 1 \}$$

(8 marks)

QUESTION FOUR (20 MARKS)

- a) Given that $h(n) = \{1, 0, 1\}$ and $h(n) = \{1, 3, 2, -3, 0, 2, -1, 0, -2, 3, -2, 1, \dots\}$, convolve the two sequences by using the following methods
 - i. Overlap and add;
 - ii. Overlap and save;
(10 marks)
- b) Given the filter

$$H(z) = \frac{1 - 0.9z^{-1} - 0.1z^{-2}}{1 + 0.3z^{-1} - 0.04z^2}$$

Realize $H(z)$ and develop the difference equation using the following form:

- i. Cascaded (series) form
- ii. Parallel form via first order-sections (10 marks)

QUESTION FIVE (20 MARKS)

- a) An analogue prototype transfer function $H(s) = \frac{s}{s-0.5}$ is used to find an equivalent low pass digital whose gain is -3dB at 400Hz and sampling frequency 4 KHz. Using bilinear transform
- i. Derive the transfer function of the digital filter
 - ii. Find the difference equation relating its output to the input sequence (12 marks)
- b) A linear-phase filter has a double zero at $z=-1$ and a conjugate pair of zeros at $z = -1$ and a conjugate pair of zeros at $0.5(1 + j\sqrt{3})$. Derive a stable, causal real impulse response $h(n)$ for this filter. Assume that the d.c gain is 1. (8 marks)

QUESTION SIX (20 MARKS)

- a)
- i Explain how circular convolution of two sequences can be obtained using FFTs and IFFTs only
 - ii Using the method in (i) and a radix-2 decimation in time FFT algorithm, find the circular convolution of the sequences.

$$X_1[n] = [3, 1, 2, 5]$$

$$X_2[n] = [1, 2, 0, 2]$$

(10 marks)

- b) Convert the analog filter $H(s) = \frac{1}{(s+1)(s+2)}$ into digital filter using the impulse invariant technique with a sampling interval of 0.02s. (10 marks)