MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN ELECTRICAL AND ELECTRONICS ENGINEERING

EEE 402: COMPLEX ANALYSIS FOR ENGINEERING

DATE: 23/3/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS

Answer question ONE and any other TWO questions

Programmable calculators are prohibited

QUESTION ONE (30 MARKS)

a) Distinguish between

i. Single valued function and many valued function (give example in each case (4 marks)

ii. Continuity in a domain and uniform continuity (2 marks)

iii. Analytic function and entire function (2 marks)

b) Define limit of a complex function f(z) and hence evaluate $\lim\limits_{z\to i} f(z) = z^2 + 2z$ (4 marks)

c) Find the image of the point (4, 3) on a z-plane under the transformation $w = 2z^2 + 3$ (4 marks)

d) Evaluate the points of discontinuity of

i. $f(z) = \dfrac{3z^2+1}{z^2+1}$ (4 marks)

ii. $f(z)=\dfrac{z^2+1}{z^4-16}$ (4 marks)

e) Show that $f(z)=-2xy+i(x^2-y^2)$ is analytic (6 marks)

QUESTION TWO (20 MARKS)

a) State the condition for integrability of $f(z)$ (2 marks)

b) State and prove the Cauchy's integral formula and give the general expression for the nth derivative of f(z) (7 marks)

c) Evaluate using Cauchy's integral formula $\oint_c \dfrac{z^2-4z+4}{z+i}dz$, where c is the circle $|z| = 2$

(4 marks)

d) Integrate z^2 along the straight line OM (direct) and along an indirect path consisting of two straight line segments OL and OM, where O is the origin, M is the point $z=3+i$ and L (3,0). Show that integral of z^2 along the two paths are equal. Hint: Sketch the region

(7 marks)

QUESTION THREE (20 MARKS)

a) Let $w=f(z)=4z+4-3i$, find the value of w which correspond to $z=-2-i$ (5 marks)

b) Evaluate the following integral $\int_C \dfrac{e^{iz}}{z^2(z-2)(z+5i)}dz$ $|z|=3$ using residue theorem.

(10 marks)

c) A point $3+bi$ on a z-plane is mapped onto the point $(11,c)$ on the w-plane by the mapping function $f(z)=2z^2+1,$ find the values of a and b (5 marks)

QUESTION FOUR (20 MARKS)

a) State three conditions for a complex function $f(z)$ to be continuous at a point $z=z_0$

(3 marks)

b) Examine the continuity of $f(z)=\dfrac{3z^4-2z^3+8z^2-2z+5}{z-i}$ (8 marks)

c) Show that $\cos^{-1}(z)=-i\ln\left(z+\sqrt{z^2-1}\right)$.Hence find all solutions to the equation $\cos z=2i$

(7 marks)

d) Use the definition of $\sin z$ and $\cos z$ to prove the following relation

$1+\tan^2 z=\sec^2 z$ (2 marks)

QUESTION FIVE (20 MARKS)

a) Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ satisfies Laplace equations (6 marks)

b) Find all the values of z such that $e^z = 1 + i\sqrt{3}$ (5 marks)

c) Find the poles of $f(z) = \frac{z^3+5z+1}{z-2}$ (5 marks)

d) Find $f(z) = u + iv$, given that $f(z)$ is analytic and $u = x^3 - 3x^2 y$ (4 marks)