MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS FOR 2017/2018 ACADEMIC YEAR

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)
BACHELOR OF (STATISTICS AND PROGRAMMING)
BACHELOR OF SCIENCE (MATHEMATICS)
BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EDUCATIOIN (ARTS)

SMA465: MEASURE AND PROBABILITY

DATE: 13/12/2017 **TIME:2.00-4.00 PM**

INSTRUCTION: Answer Question **ONE** and Any Other **TWO** Questions

QUESTION ONE

(a) Define the following terms

(i) Set function (1 mark)

(ii) Field (1 mark)

(iii) $\sigma -$ field (1 mark)

(b) Let A be a set with n elements and P^A be the power set of A. Show that the number of subsets of A contained in P^A is 2^n (5 marks)

(c) Determine whether the following sequences are monotone increasing or monotone decreasing, hence state their limits

(i) $A_n = \{w: 2 - \frac{2}{n} \leq w \leq 5 + \frac{1}{n},\ n \in \mathbb{R}\}$ (3 marks)

(ii) $A_n = \{w: 4 + \frac{2}{n} \leq w \leq 10 - \frac{4}{n},\ n \in \mathbb{R}\}$ (3 marks)

(d) Show that $I_{AB} + I_{AUB} = I_A + I_B$ where I_A and I_B are indicator function of A and B respectively. (4 marks)

(e) Define a measure on a measurable space. Show that probability is a measure (5 marks)

(f) Prove that $E(cX) = cE(X)$ (3 marks)

(g) State, without prove, the Borel-Cantelli Lemma (4 marks)

QUESTION TWO (20 MARKS)

(a) Differentiate between monotone increasing and monotone decreasing sequence (3 marks)

(b) Prove that a monotone field is a σ − field and vice-versa (6 marks)

(c) Consider the set ฿ whose elements are intervals $(x, \infty) \in \mathbb{R}$. If $A_1 = (u, \infty)$ and $A_2 = (v, \infty)$. Is ฿, closed under ฿

(i) Intersection (2 marks)

(ii) Union (2 marks)

(iii) Complementation (2 marks)

(iv) Is it a field? (1 mark)

(d) Distinguish between finite and σ − finite measure (4 marks)

QUESTION THREE (20 MARKS)

(a) Let $I_A = \begin{cases} 1, if\ w \in A \\ 0,\ if\ w \in A^c \end{cases}$ and $I_B = \begin{cases} 1, if\ w \in B \\ 0,\ if\ w \in B^c \end{cases}$ be indicator functions of A and B respectively. Show that

(i) $I_{A^c} = 1 - I_A$ (3 marks)

(ii) $I_{AB} = I_A I_B$ (3 marks)

(iii) $I_{A \cup B} = I_A + I_B - I_{A \cap B}$ (4 marks)

(b) Show that inverse mapping preserve the following set relations

(i) $X^{-1}\left(\bigcap_k B'_k\right) = \bigcap_k X^{-1}(B'_k)$ (5 marks)

(ii) $X^{-1}\left(\bigcup_k B'_k\right) = \bigcup_k X^{-1}(B'_k)$ (5 marks)

QUESTION FOUR(20 MARKS)

(a) What is convergence in probability (2 marks)

(b) Prove that the necessary and sufficient condition for convergence in probability is that

$$E\left[\frac{|X_n|}{1+|X_n|}\right] \rightarrow 0$$ (6 marks)

(c) State and prove Chebyshev's inequality (6 marks)

(d) State and prove the weak law of large numbers. (6 marks)

QUESTION FIVE (20 MARKS)

(a) Differentiate between a measure and measurable space (4 marks)

(b) Discuss axiomatic concept of probability (6 marks)

(c) Given a class $\{A_i, i = 0,1,2,\ldots,n\}$ of n sets and there exist a class $\{B_i, i = 0,1,2,\ldots,n\}$ such that B_i's are disjoint, show that

$$\bigcup_{i=1}^{n} A_i = \sum_{i=1}^{n} B_i$$

(6 marks)

(d) Show that $E[X \mp Y] = E[X] \mp E[Y]$ (4 marks)