MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION ARTS

SMA 305: COMPLEX ANALYSIS I.

DATE: 13/4/2023 **TIME: 2:00 – 4:00 PM**

***INSTRUCTIONS**: answer Question 1 and any other 2 Questions.*

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Find all the 3rd roots of -8 and plot these values in the argand diagram (5 marks)

b) Represent $z = 4 - 4\sqrt{3}i$ as a complex number in polar form (5 marks)

c) Given $e^{iy} = cosy + isiny$; $e^{-iy} = cosy - isiny$, find the representation for $cotx$ (5 marks)

d) Find $\lim_{z \to 2i} \frac{\bar{z} + z^2}{1 - \bar{z}}$ (5 marks)

e) Verify that the function $f(z) = z^3 + 1$ satisfy the Cauchy-Riemann equations (5 marks)

f) When is a complex valued function said to be analytic? Is the function $f(z) = |z|^2$ analytic in the complex plane? (5 marks)

QUESTION TWO (20 MARKS)

a) Find all complex numbers for which $cosz = 2.5$ (7 marks)

b) Find the image of the line $x = 2$ under the mapping $w = \frac{1}{z}$. (7 marks)

c) Express the polar form of $\left[\frac{2+i2\sqrt{3}}{1+i}\right]$ (6 marks)

QUESTION THREE (20 MARKS)

a) Evaluate $\int_c \frac{1-2z}{z(z-1)(z-3)} dz$ where c is a circle $|z| = \frac{3}{2}$ using Cauchy's residue theorem (7 marks)

b) Let C be the curve $y = x^3 - 3x^2 + 4x - 1$ joining the points $(1,1)$ and $(2,3)$. Find the value of $\int_c (12z^2 - 4iz)dz$ (7 marks)

c) Evaluate $\oint_c \frac{dz}{z^2+9}$ where c is $|z + 3i| = 2$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Expand $f(z) = logz$ about $z_0 = 1$ by the Taylor series and determine the region of convergence (7 marks)

b) Find the Laurent series expansion for $f(z) = \frac{1}{1+z}$ for $|z| < 1$ (6 marks)

c) Calculate the residues of $f(z) = \frac{1}{z^2+2z+5}$ at the poles (7 marks)

QUESTION FIVE (20 MARKS)

a) Show that $u = x^3 - 3xy^2$ is harmonic, find their harmonic conjugates and express $u + iv$ as an harmonic function of z (7 marks)

b) Derive the Cauchy-Riemann equations. (7 marks)

c) Show that $f(z) = e^x cosy + ie^x siny$ is an entire function (6 marks)