MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

FOURTH YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)
BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)
BACHELOR OF SCIENCE (MATHEMATICS)
BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EDUCATION (ARTS)

SMA 463-TIME SERIES ANLYSIS

DATE: 22ND JANUARY ,2025 **TIME: 8.30 AM - 10.30 AM**

INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

(a) Explain the following terms as applied as in term series analysis

(i) Stationarity. (2marks)

(ii) Filtering (2marks)

(b) Differentiate between causality and invertibility as used in time series. (4marks)

(c) State and explain the TWO main models of time series analysis (4marks)

(d) Let $X_t: t = 0, \mp 1, \mp 2, \ldots.$ be a stochastic process given by $X_t = a + bt + e_t$ where e_t is a sequence of independent random variables distributed with $\mu = 0, and\ \sigma^2$ as $e_t \sim N(0, \delta^2)$.Show that $X_t = (X_t - X_{t-1})$ is stationary (5marks)

(e) (i) Let $X_t: t = 0, \mp 1, \mp 2, \ldots.$ be a stationary time series ,define $f_x(\lambda)$ the spectral density function of time series (3marks)

(ii) Let $\{e_t\}$ be purely random process, determine $f_e(\lambda)$ the spectral function of $\{e_t\}$ (4marks)

(f) Consider two MA(1) processes P and Q given by

$$P: X_t = e_t - 3e_{t-1}$$
$$Q: X_t = e_t - \frac{1}{3}e_{t-1}$$

(i) Obtain the autocorrelation function for the two processes. (4marks)

(ii) Explain whether moving average process uniquely identified from a given autocorrelation. (2 marks)

QUESTION TWO (20 MARKS)

(a) Consider the autoregressive process of order 2 given by $X_t = \propto_1 x_{t-1} + \propto_2 x_{t-2} + e_t$ where (e_t) is a purely random process. Derive the Yule-Walker equation and hence its general solution. What are the values of $\propto_1$ and $\propto_2$ for which the process is stationary.

(8marks)

(b) Define a moving average process of order q, [MA(q)] (2marks)

hence:

(i) Determine its mean and variance. (4marks)

(ii) Obtain its covariance and autocorrelation functions. (4marks)

(iii) State giving reason, whether it is a second order stationary. (2marks)

QUESTION THREE (20 MARKS)

(a) Explain the following statements

(i) Time series is stationary in the weak sense. (3 marks)

(ii) Time series is stationary in the strict sense (3marks)

(b) Determine if the following ARMA process is causal and if it is invertible (6marks)

$$X_t + 1.6X_{t-1} = Z_t - 0.4Z_{t-1} + 0.04Z_{t-2}$$

(c) Consider the first-order moving average process defined by the equation

$X_t = \alpha X_{t-1} + Z_t$, Using the backward shift operator **B**, the process can be written as

$(1 - \alpha B)X_t = Z_t$,Show that the $Var(X_t) = \frac{\sigma_z^2}{1-\alpha^2}$ (8marks)

QUESTION FOUR (20 MARKS)

(a) State and explain the components of time series analysis. (8marks)

(b) Show that the autocorrelation function (acf) of the stationary second order AR process

$X_t = \frac{1}{12}X_{t-1} + \frac{1}{12}X_{t-2} + Z_t$ where $Z_t \sim iid\ (0, \sigma_z^2)$ is given by

$$\rho(h) = \frac{45}{77}\ (\frac{1}{3})^{|h|} + \frac{32}{77}(-\frac{1}{4})^{|h|} \qquad h = 0, \pm 1, \pm 2, ...$$

(12marks)

QUESTION FIVE (20 MARKS)

(a) (i) Define the spectral density function of a time series X_t and show that it can be expressed in terms of cosines. (4marks)

(ii) Derive the spectral density function of a purely random process. (4marks)

(b) Consider a finite MA process defined as;

$X_t = \sum_{j=0}^{m} \beta_{i,} e_{t-j}$, where e_t is a purely random process and let $f_x(\lambda)$ be the spectral density function of X_t

Show that $f_x(\lambda) = \frac{\delta^2}{2\pi(1+2\beta_1 cos\lambda+{\beta_1}^2)}$ (12marks)