



# MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 107: ENGINEERING MATHEMATICS IV

DATE: 12/4/2023

TIME: 11:00 – 1:00 PM

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## INSTRUCTIONS TO CANDIDATES.

Answer Question One and Any Other Two Questions

### QUESTION ONE (COMPULSORY) (30 MARKS)

- a) From the first principle find the first derivative of  $y = 4x^3$  (3 marks)
- b) The parametric equations of a function are given by  $y = \cos 2t$ ,  $x = 2\sin t$
- i.  $\frac{dy}{dx}$  (2 marks)
- ii.  $\frac{d^2y}{dx^2}$  (3 marks)
- c) Evaluate
- i.  $\int \frac{3x^2}{x^3-4} dx$  (3 marks)
- ii.  $\int \frac{x+1}{x^2-3x+2} dx$  (3 marks)
- iii.  $\int x^2 \ln x dx$  (3 marks)
- d) Determine  $\frac{dy}{dx}$  when  $x = -1$  given  $y = x^x$  (3 marks)
- e) Find the equation of the tangent to the curve  $y = x^2 - x - 2$  at  $(1, -2)$  (3 marks)
- f) Given  $y = 4x^2 - x$  determine the approximate small change in  $y$  if  $x$  changes from 1 to 1.02. (3 marks)
- h) Find  $\frac{dy}{dx}$  given
- i.  $y = \cos(5x^2 + 2)$  (2 marks)
- ii.  $y = (3x - 1)^9$  (2 marks)

### QUESTION TWO (20 MARKS)

- a) Determine the area enclosed by the two curves

$y = x^2$  and  $y = 8x$ . if this area is rotated  $360^\circ$  about the x-axis, determine the volume of the solid of revolution produced. (10 marks)

- b) The current  $i$  in an electric circuit containing resistance  $R$  and inductance  $L$  in series with a constant voltage source  $E$  given by the differential equation

$$E - L \left( \frac{di}{dt} \right) = Ri$$

Solve the equation and find  $I$  in terms of time  $t = 0, i = 0$  (10 marks)

### QUESTION THREE (20 MARKS)

- a) Use Simpson's rule with four intervals to evaluate  $\int_1^3 \frac{2}{\sqrt{x}} dx$  (6 marks)

- b) Use mid-ordinate rule with six intervals to evaluate  $\int_0^{2.4} e^{\frac{-x^2}{3}} dx$  (8 marks)

- c) Locate the turning point on the following curve and determine its nature

$$y = 4\theta + e^{-\theta} \quad (6 \text{ marks})$$

### QUESTION FOUR (20 MARKS)

- a) Evaluate  $\int_2^3 \frac{x^3 - 2x^2 - 4x - 4}{x^2 + x - 2} dx$  (6 marks)

- b) Evaluate  $\int_0^{\pi/4} 4\cos^4\theta d\theta$  (5 marks)

- c) The equation of the tangent drawn to the parabola  $x = 2t^2, y = 4t$  at the point  $t$ . (5 marks)

- d) Show that the derivative of  $\sec ax = a \sec(ax) \tan(ax)$  (4 marks)

### QUESTION FIVE (20 MARKS)

- a) Differentiate the following with respect to  $x$

i.  $\sin 3x$  (2 marks)

ii.  $4\ln x$  (2 marks)

- b) Find  $\frac{dy}{dx}$  in terms of  $X$  and  $y$  given  $4x^2 + 2xy^3 - 5y^2 = 0$  and evaluate  $\frac{dy}{dx}$  when  $x=2$  and  $y=2$  (6 marks)

- c) Use trapezoidal rule to evaluate  $\int_0^{\pi/2} \frac{1}{1+\sin x} dx$  using six intervals and give your answer to 4 significant figures. (5 marks)

- d) Determine the area enclosed between the curves  $y = x^2 + 1$  and  $y = 7 - x$  (5 marks)