



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 103: ANALYTICAL GEOMETRY

DATE: 25/3/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30 MARKS)

- a) A point $R(x, y)$ divides the line through the points $A(4, -9)$ and $B(-5, 25)$ externally in the ratio $3 : 2$. Calculate the coordinates of $R(x, y)$ (3 marks)
- b) Calculate the equation of the straight line which passes through the point of intersection of the lines $4x - 3y + 19 = 0$, $12x - 5y + 3 = 0$ and makes an intercept of $+2$ on the x -axis (5 marks)
- c) Prove that $\cos 4\theta = 8\cos^4 \theta - 8\cos^2 \theta + 1$ (4 marks)
- d) Calculate the polar equation of the circle center $C(5, 8)$ and radius $r = 10$ units (5 marks)
- e) A parabola $y = ax^2 + bx + c$ passes through the points $(1, 0)$, $(2, 2)$, and $(3, 10)$. Determine its equation. (5 marks)
- f) Work out the Cartesian equation for the curve $r \cos \left(\theta - \frac{\pi}{3} \right) = 3$ (4 marks)
- g) Determine the polar equation of the ellipse $x^2 + 9y^2 = 9$ (4 marks)

QUESTION TWO (20MARKS)

- a) Determine the equation of a circle through the points $A(6,-1)$, $B(-2,5)$, $C(-6, -7)$, sketch the circle (6 marks)
- b) Calculate the equation of the circle which is tangent to the x-axis and concentric with the circle $2x^2 + 2y^2 - 11x + 6y - 8 = 0$ (6 marks)
- c) Determine the polar equation of the circle whose center has the Cartesian coordinates $C(5,8)$ and radius $r = 10cm$ (5 marks)
- d) Calculate the distance from the point $(5,8)$ to the circle $x^2 + y^2 + 2x + 6y + 3 = 0$ (3 marks)

QUESTION THREE (20MARKS)

- a) Solve for x in $\sin 3x + \sin x = 0$ (4 marks)
- b) Determine the value of x for which $2 \cos x - 2\sqrt{3} \sin x = 3$ (5 marks)
- c) Show that
- i. $\frac{\sin 4\theta + \sin 2\theta}{\cos 4\theta + \cos 2\theta} = \tan 3\theta$ (4 marks)
- ii. $\sin 4\theta = 4 \cos^3 \theta \sin \theta - 4 \sin^3 \theta \cos \theta$ (3 marks)
- d) Given that $\cos 2x = \frac{1}{8}$, calculate the value of $\cos x$ (4 marks)

QUESTION FOUR (20 MARKS)

- a) Sketch the curve and label axes, foci and vertices described by the equation $\frac{x^2}{9} + \frac{y^2}{4} = 1$ (6 marks)
- b) Calculate the point on the parabola $y^2 - 12y - 14x + 162 = 0$ through which the tangents which meet at $Q(2,4)$ pass (6 marks)
- c) Determine the equation of the tangent to the parabola $x^2 - 22x + 24y + 169 = 0$ at the point whose focal distance is 11 units (8 marks)

QUESTION FIVE (20 MARKS)

- a) Sketch the curve and label axes, foci and vertices described by the equation

$$\frac{x^2}{25} - \frac{y^2}{16} = -1 \quad (12 \text{ marks})$$

- b) Calculate the angle between the lines

$$y = 2x + 5 \quad \text{and} \quad y = 12x - 1 \quad (4 \text{ marks})$$

- c) Calculate the normal equation of the line with $\rho = 10 \text{ units}$, $\omega = 60^\circ$ (4 marks)