MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SAC 101: FUNDAMENTALS IN ACTUARIAL MATHEMATICS

DATE: 15TH JANUARY, 2025 **TIME: 8.30 AM – 10.30 AM**

INSTRUCTIONS:
*Answer question **ONE (Compulsory)** and **any other TWO** questions*

QUESTION ONE (30 MARKS)

a) Define the following notations
   i. T_x **(2 marks)**
   ii. δ **(2 marks)**

b) An investment of £5,000 in an account accumulated to £10,500 after 10 years.
   i. State the accumulation factors A (0,10). **(2 marks)**
   ii. Based on part (i) Find the annual compound interest rate which would give the accumulation factor. **(3 marks)**

c) Show that: $\ddot{a}_{\overline{n}|} = \frac{1-v^n}{d}$. **(5 marks)**

d) Find the annual effective interest rate equivalent to a nominal interest rate of 12% p.a convertible four monthly. **(5 marks)**

e) Calculate the present value on 1 September 2002 of payments of £280 due on 1 September 2004 and £360 due on 1 March 2005. Interest is 15% pa effective. **(5 marks)**

f) Define and give the equation for evaluating the sinking fund. **(6 marks)**

QUESTION TWO (20 MARKS)

a) On the basis of E.L.T. No. 15 - Males, find the probability that a life aged 30 will

   i. Die between his 40th birthday and his 50th birthday**.** **(2 marks)**

   ii. Die either between exact ages 35 and 45 or between exact ages 70 and 80**.**

(3 marks)

b) A life insurance company issues an annuity to a life aged 60 exact. The purchase price is £200,000. The annuity is payable monthly in advance and is guaranteed to be paid for a period of 10 years and for the whole of life thereafter.

   Calculate the annual annuity payment.

   Basis:

   i) Mortality AM92 ultimate.

   ii) Interest 6% per annum **(15 marks)**

QUESTION THREE (20 MARKS)

a) Define the following terminologies:

   i. Amortization table. **(2 marks)**

   ii. Retrospective method **(2 marks)**

b) A loan is being amortized with a series of 20 annual payments. if the amount of principal in the third payment is £200, what is the amount of principal in the last 3 payments? (the interest rate is 4 %) **(16 marks)**

QUESTION FOUR (20 MARKS)

The market price of a £ 1000 par value bond carrying a coupon rate of 14% and maturing after 5 years is £1050.

a) What is the yield to maturity of this bond? **(12 marks)**

b) What is the approximate yield to maturity? **(2 marks)**

c) What will be the realized yield to maturity if the re-investment rate is 12%?
(6 marks)

QUESTION FIVE (20 MARKS)

a) Show that: $i^{(p)} = p[(1+i)^{\frac{1}{p}} - 1]$ **(5 marks)**

b) Project R delegates all the development work to outside companies. The estimated cash flows for project R are: (last column indicates expenditure)

Beginning of year 1	150,000 dollars	Contractor fees
Beginning of year 2	250,000 dollars	Contractor fees
Beginning of year 3	250,000 dollars	Contractor fees
End of year 3	1,000,000 dollars	Sales

Table 1

Project S carries out all the development work in-house by purchasing the necessary equipment and using the company's own staff. The cash flows for the project S are: (last column indicates expenditure)

Beginning of year 1	325,000 dollars	New equipment
Throughout year 1	75,000 dollars	Staff cost
Throughout year 2	90,000 dollars	Staff cost
Throughout year 3	120,000 dollars	Staff cost
End of year 3	1,000,000 dollars	Sales

Table 2

The staff cost can be estimated to be paid uniformly throughout the year.

i) Evaluate the NPV for the project R and S using a risk discount rate of 10% pa? **(13 marks)**

ii) Using the NPV as a criterion, which project is preferable? **(2 marks)**