



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS & PROGRAMMING.

BACHELOR OF SCIENCE IN COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 330-NUMERICAL ANALYSIS

DATE: 25/3/2021

TIME: 8.30-10.30 AM

INSTRUCTION TO CANDIDATES

ANSWER QUESTION ONE AND ANY TWO OTHER QUESTIONS

QUESTION ONE COMPULSORY (30 MARKS)

- a) Estimate the error in z if $z = \frac{x}{y} + 2x$ given that $x = 3.213$ and $y = 6.172$ (2 marks)
- b) Show that the operators μ and E commute. (3 marks)
- c) Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method. (5 marks)
- d) prove that the forward difference of the quotient of two functions is given by
- $$\Delta \left(\frac{f(x)}{g(x)} \right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)} \quad (5 \text{ marks})$$
- e) Set up Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (7 marks)
- f) find a root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ with an accuracy of 10^{-4} (8 marks)

QUESTION TWO (20 MARKS)

- a) If E, μ and δ denote shift, average and central difference operators, in analysis of data with equal spacing h , prove the following

$$E^{1/2} = \mu + \frac{\delta}{2} \quad (2 \text{ marks})$$

- b) Starting with $x_0 = 3$, find an approximate root of $f(x) = x^3 - 3x - 5$ correct to 3dp by Newton-Raphson method upto 4 iterations (5 marks)
- c) Solve the equation $x^3 = \sin x$. Considering various $\phi(x)$, discuss the convergence of the solution. (5 marks)
- d) using the values given in the following table, find $\cos 0.28$ by linear interpolation and by quadratic interpolation and compare the results with the value 0.96106 (exact to 5D) (8 marks)

QUESTION THREE (20 MARKS)

- a) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(4 marks)

- b) Construct the forward difference table, where $f(x) = \frac{1}{x}, x = 1(0.2)2.4$ (4 marks)
- c) Find the cubic polynomial which takes the following values; $f(1) = 24, f(3) = 120, f(5) = 336$, and $f(7) = 720$. hence or otherwise, obtain the value of $f(8)$. (6 marks)
- d) Evaluate $\int_0^1 e^{-x^2} dx$ by means of Trapezoidal rule with $n = 10$ (6 marks)

QUESTION FOUR (20 MARKS)

a) Find the percentage error in the quotient $\frac{25.4}{12.37}$ (4 marks)

b) Find $\int_0^{10} \frac{1}{1+x^2} dx$ using Simpson's one third rule. (7 marks)

c) Find the interpolating polynomial by Newton's divided formula for the following table and then calculate $f(2.1)$.

x	0	1	2	4
$f(x)$	1	1	2	5

(9 marks)

QUESTION FIVE (20 MARKS)

a) Construct the difference table for the following data tabulated function:

x	1.0	1.2	1.4	1.6	1.8	2.0
$f(x)$	1.5431	1.8107	2.1509	2.5775	3.1075	3.7622

Use suitable Newton's interpolating polynomial to estimate $f(1.1)$ and $f(1.9)$ (10 marks)

b) Use the equation $f(x) = 3x^3 - 6x - 2$ to show that $x_n = \sqrt[3]{\frac{6x_n + 2}{3}}$, use $x_0 = 1$ get linear iterates to 5 dp (10 marks)