



MACHAKOS UNIVERSITY

University Examinations 2021/2022 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 300: DIFFERENTIAL EQUATIONS

DATE: 23/8/2022

TIME: 2.00-4.00 PM

INSTRUCTION:

Answer Question One and Any Other Two Questions

Unit code: SAC 300

Unit: Differential Equations

QUESTION ONE (30 MARKS)

- a) Solve $\frac{dy}{dx} - 3y = 0$ (3 marks)
- b) Solve $2xy \, dx + (x^2 - 1)dy = 0$ (4 marks)
- c) Solve the equation $2t(t - \frac{d\phi}{dt}) = 5$ given $\phi = 2$ when $t = 1$ (5 marks)
- d) Determine the particular solution of $\frac{d\theta}{dt} = 2e^{3t-2\theta}$ given that $t = 0$ when $\theta = 2$ (5 marks)
- e) Find the particular solution of the homogeneous equation $x \frac{dy}{dx} = \frac{x^2+y^2}{y}$ given the boundary condition that $y = 4$ when $x = 1$ (6 marks)
- f) Solve $\frac{\partial u}{\partial t} = 2t \cos \theta$ given that $u = 2t$ when $\theta = 0$ (4 marks)
- g) Determine the general solution of $\frac{\partial u}{\partial y} = 4ty$ (3 marks)

QUESTION TWO (20 MARKS)

- a) Solve the following differential equations

$$x \frac{dy}{dx} = 3 - y \quad (3 \text{ marks})$$

$$x \left(\frac{dy}{dx} + 1 \right) = x^3 - 2y \quad (5 \text{ marks})$$

- b) i. The variation of Resistance, R ohms of an aluminum conductor with temperature $\theta^\circ C$ is given by $\frac{dR}{d\theta} = \gamma R$ where γ is the temperature coefficient of resistance of aluminum. If $R = R_0$ when $\theta = 0^\circ C$ solve the equation for R (6 marks)
- ii. If $\gamma = 38 \times 10^{-4}/^\circ C$ determine the resistance of an aluminium conductor at 50° correct to 3-significant figures when the resistance at $0^\circ C$ is 240Ω (6 marks)

QUESTION THREE (20 MARKS)

- a) Use the Frobenius method to determine the general power series solution of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + (1 - x)y = 0 \quad (15 \text{ marks})$$

- b) Solve the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 4 \quad (5 \text{ marks})$$

QUESTION FOUR (20 MARKS)

- a) Use Lagranges method to solve

$$p + 3q = 5z + \tan(y - 3x) \quad (10$$

marks)

- b) Find the Wronskian for

i. $y_1 = e^{4x}$, $y_2 = xe^{4x}$ (5 marks)

ii. $y_1 = \cos 3x$, $y_2 = \sin 3x$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Solve the differential equation

$$2 \frac{d^2y}{dx^2} - 11 \frac{dy}{dx} + 12y = 3x - 2 \quad (7 \text{ marks})$$

- b) Find the general solution of $y'' + 3y' + 2y = 0$ using the Wronskian method (6 marks)

- c) Given $(y^2 + z^2 - x^2)p - 2xyq - 2xz = 0$ solve using Lagranges method (7 marks)