MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

………..YEAR …… SEMESTER SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

BACHELOR OF SCIENCE IN MATHEMATICS

BACHELOR OF EDUCATION

SMA 302: GROUP THEORY

DATE: **TIME:**

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE 30 MARKS (COMPULSORY)

a) Define the following terms

i) Subgroup

ii) Cyclic group.

iii) Permutation

iv) A transposition

v) Normal subgroup (5 marks)

b) Construct the multiplication table for A_3 (5 marks)

c) Prove that every cyclic group is abelian. (5 marks)

d) Define a group (5 marks)

e) Prove that any two right (left) cosets of H are disjoint. (5 marks)

f) Construct the addition table for Z_{16} and show that $\{0,5\}\, and \{0,1\}$ are not the proper subgroups and construct its lattice diagram. (5 marks)

QUESTION TWO (20 MARKS)

a) Prove that the group (G) of prime order is cyclic. (5 marks)

b) Prove that the right cosets of H in G are one to one correspondence. (5 marks)

c) Prove that if G is a group and H non empty subset of G then H is a subgroup of G iff for $a, b \in H;\ ab^{-1} \in H.$ (5 marks)

d) Prove that if $b \in Ha$ then $Hb = Ha$ (5 marks)

QUESTION THREE

a) State and prove the LaGrange's' Theorem (6 marks)

b) Prove that if H is a subgroup of G. then each coset has the same number of elements as H. (5 marks)

c) Define a function $f: G \to G^1$ by $G(x) = axa^{-1}$ show that f is isomorphism. (4 marks)

d) Find all the subgroups of D_3 and construct its lattice diagram (5 marks)

QUESTION FOUR (20 MARKS)

a) Prove that every subgroup of index two is normal. (6 marks)

b) Let G denotes the set of all ordered pair of real numbers with non-zero 1st component of the binary operation$*$. Then $*$ is defined on G by the rule $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G,*)$ is a group. (8 marks)

c) Show that every subgroup of an abelian group is normal. (6 marks)

QUESTION FIVE (20 MARKS)

a) Let H be a normal subgroup of G. Denote the set of all left cosets $\{\{aH | a \in G\}\}$ by ${}^{G}/_{H}$ and define $*$ on ${}^{G}/_{H}$ by for all $aH, bH \in {}^{G}/_{H}$ $(aH) * (bH) = abH$ then $\left({}^{G}/_{H}, *\right)$ is a group. (5 marks)

b) Prove that if A be a non-empty set and let S_A be the collection permutations of A. Then S_A is a group under permutation multiplication. (5 marks)

c) Prove that the identity and the inverse of a group is unique (5 marks)

d) Find all the left cosets of A_3 (5 marks)