MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FOURTH YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS)

SMA 433: PARTIAL DIFFERENTIAL EQUATIONS II

DATE: 20/1/2021 **TIME: 8.30-10.30 AM**

INSTRUCTIONS TO CANDIDATES

Attempt question ***one (compulsory***) and any other ***two questions***.

QUESTION ONE (30 MARKS)

a) Define a second order linear partial differential equation (4 marks)

b) State the short form a second order partial differential equation (3 marks)

c) State the three classes of the second order partial differential equation (4 marks)

d) Consider the equation $2u_{xx} + 3u_{xy} + u_{yy} = 0$.

i. Classify the equation (4 marks)

ii. Determine its characteristic equation (5 marks)

iii. Transform the equation to canonical form (7 marks)

iv. Solve the equation (7 marks)

QUESTION TWO (20 MARKS)

Prove the general equation $A\dfrac{\partial^2 u}{\partial x^2} + B\dfrac{\partial^2 u}{\partial x \partial y} + C\dfrac{\partial^2 u}{\partial y^2} + D\dfrac{\partial u}{\partial x} + E\dfrac{\partial u}{\partial y} + Fu = 0$

Where A, B, C, D, E, F are real constants such that $A \neq 0$.

QUESTION THREE (20 MARKS)

Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$

Given that $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$.

QUESTION FOUR (20 MARKS)

Determine the characteristic of the Tricomi equation $y u_{xx} + u_{yy} = 0$ in the lower-half plane $y < 0$. Put this equation into canonical form in the upper half- plane $y > 0$.

QUESTION FIVE (20 MARKS)

a) An infinitely long string having one end at $x = 0$ is initially at rest on the $x - axis$. At $t = 0$;the end $x = 0$ begins to move along the $u - axis$ in a manner described by $u(0,t) = a\cos\sigma t$. Determine the displacement $u(x,t)$ of the string at any point at any time. (10 marks)

b) Using Mong'es method obtain the integral of $q^2 r - 2pqs + p^2 t = 0$ in the form $y + xf(z) = F(z)$ (10 marks)