



MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

DECEMBER SESSION EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 330: NUMERICAL ANALYSIS 1

DATE: 25/4/2019

TIME:

INSTRUCTION: Answer Question *ONE* which is compulsory and any other *TWO* Questions

QUESTION ONE COMPULSORY (30 MARKS)

- a) Show that the operators Δ and E commute. (2 marks)
- b) If 0.182 is the approximate value of $\frac{2}{11}$ find the absolute, relative and percentage errors. (4 marks)
- c) Use Lagrange's formula to prove that $y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{16}[(y_3 - y_1) - (y_{-1} - y_{-3})]$ (5 marks)
- d) Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$, by bisection method. (5 marks)
- e) How accurately should the length and the time of vibration of a pendulum should be measured in order that the computed value of g be correct to 0.01% (7 marks)

QUESTION TWO (20 MARKS)

- a) Prove that the forward difference of the quotient of two functions is given by

$$\Delta\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)\Delta f(x) - f(x)\Delta g(x)}{g(x+h)g(x)} \quad (5 \text{ marks})$$

- b) Set up a Newton iteration for computing the square root of a given positive number. Using the same find the square root of 2 exact to six decimal places. (7 marks)
- c) Find a root of the equation $x^3 + x^2 - 1 = 0$ on the interval $[0,1]$ with an accuracy of 10^{-4} (8 marks)

QUESTION THREE (20 MARKS)

- a) Construct the forward difference table, where $f(x) = \frac{1}{x}$, $x = 1(0.2)2.4$ (4 marks)
- b) Find $\int_0^{10} \frac{1}{1+x^2} dx$ using Simpson's one third rule. (7 marks)
- c) Find the interpolating polynomial by Newton's divided formula for the following table and then calculate $f(2.1)$.

x	0	1	2	4
$f(x)$	1	1	2	5

(9 marks)

QUESTION FOUR (20 MARKS)

- a) Using Aitken's scheme and the following values evaluate $\log_{10} 301$

x	300	304	305	307
$\log_{10} x$	2.4771	2.4829	2.4843	2.4871

(4 marks)

- b) Find the positive solution of the transcendental equation $2\sin x = x$ (5 marks)
- c) Find the cubic polynomial which takes the following values;
 $f(1)=24$, $f(3)=120$, $f(5)=336$, and $f(7)=720$. hence or otherwise, obtain the value of $f(8)$. (5 marks)

- d) Evaluate $\int_0^1 e^{-x^2} dx$ by means of Trapezoidal rule with $n = 10$ (6 marks)

QUESTION FIVE (20 MARKS)

- a) Derive the Newton-Raphson iterative formula

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, n \geq 1$$

For the solution of the non-linear equation $f(x) = 0$. (10 marks)

- b) Use the method in a.) to find the root of the equation $\tan x = x$ near $x = 4.5$ correct to four decimal places. (10 marks)