



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR
BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SST 103: LINEAR ALGEBRA

DATE: 26/3/2021

TIME: 11.00-1.00 PM

INSTRUCTIONS

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) (COMPULSORY)

a) Find the cross product of the following vectors $(2, 4, -1)$ and $(3, 6, 1)$ (3 marks)

b) Reduce the following matrixes to be reduced echelon form

$$A = \begin{pmatrix} 3 & 4 & -1 & 1 \\ 1 & -1 & 3 & 2 \\ 4 & -3 & 11 & 2 \end{pmatrix} \quad (3 \text{ marks})$$

c) Solve the following simultaneous equation using inverse matrix method

$$2x - y = 4$$

$$3x + 2y = 6 \quad (3 \text{ marks})$$

d) If $\begin{vmatrix} y & 5 \\ y & 2y + 1 \end{vmatrix} = 4$ find y (3 marks)

e) Calculate the cross product of the vectors $\vec{u} = (1, 2, 3)$ and $\vec{v} = (-1, 1, 2)$. (3 marks)

f) Find the inverse of the following matrix

$$\begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 2 \\ 3 & 3 & 1 \end{bmatrix} \quad (4 \text{ marks})$$

- g) Show that the vector (11,3,-8) is a linear combination of the (1,1,0) and (2,1,-1)
- h) Calculate the x and y values for the vector (x, y, 1) that is orthogonal to the vectors (3, 2, 0) and (2, 1, -1). (4 marks)
- i) Determine if (1,2,3,1), (2,2,1,3) and (-1,2,7,3) is linearly dependent (4 marks)
- j) Find the angle between $u = 2i + 2j + 2k$ and $v = i + j + k$ (3 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Find the rank of the following matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ 3 & -3 & 2 \\ 2 & 0 & 3 \end{bmatrix} \quad (4 \text{ marks})$$

- b) Transpose the following matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 4 & 7 \end{pmatrix} \quad (2 \text{ marks})$$

- c) Find the minors, cofactors and adjoint of the following matrix

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 3 & 2 \\ 1 & 3 & 1 \end{bmatrix} \quad (4 \text{ marks})$$

- d) Find the inverse of the following matrices

$$\begin{pmatrix} \cos \theta & \sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (5 \text{ marks})$$

- e) Reduce the following matrix into echelon form

$$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \end{pmatrix} \quad (5 \text{ marks})$$

QUESTION THREE (20 MARKS)

- a) Solve by crammer's Rule

$$x - 2y + 3z = 10$$

$$3x - 6y + z = 22$$

$$-2x + 5y - 2z = -18 \quad (5 \text{ marks})$$

- b) Determine the value of 'a' so that the following systems in unknown x , y and z has

i. No solution

ii. More than one solution

iii. Unique solution

$$x - 3z = -3$$

$$2x + ay - z = -2$$

$$x + 2y + az = 1 \quad (6 \text{ marks})$$

- c) Solve the following system of equation by elimination method

$$x_1 + x_2 + x_3 = 5$$

$$x_1 + 2x_2 + 3x_3 = 10$$

$$2x_1 + x_2 + x_3 = 6 \quad (4 \text{ marks})$$

- d) Solve the simultaneous equation using the gauss-Elimination method

$$2x_1 - 4x_2 + 6x_3 = 20$$

$$6x_1 - 12x_2 + 2x_3 = 44$$

$$-4x_1 + 10x_2 - 4x_3 = -36 \quad (5 \text{ marks})$$

QUESTION FOUR (20 MARKS)

- a) Define a vector space (6 marks)
- b) Show that $W = \{(x, y) / x = 2y\}$ is a subspace for $\mathbb{R}^2$. (4 marks)
- c) Prove that the diagonals of a rhombus are perpendicular. (5 marks)
- d) Find the parametric and the symmetric equations of the line passing through the point $(2, 3, -4)$ and parallel to the vector $(3, 5, -6)$ (5 marks)

QUESTION FIVE (20 MARKS)

- a) Show that u , v , and w are linearly independent

$$u = (6, 2, 3, 4)$$

$$v = (0, 5, -3, 1)$$

$$w = (0, 0, 7, -2) \quad (5 \text{ marks})$$

- b) Write $E = \begin{pmatrix} 3 & 1 \\ 1 & -1 \end{pmatrix}$ as a linear combination of the matrices

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \text{ and } C = \begin{pmatrix} 0 & 2 \\ 0 & -1 \end{pmatrix} \quad (5 \text{ marks})$$

- c) Calculate the value of k for the vectors $\vec{u} = (1, k)$ and $\vec{v} = (-4, k)$ knowing that they are orthogonal. (5 marks)

- d) Show that the vectors $u=(1,-1,0)$ $v=(1,3,-1)$ and $w=(5,3,-2)$ are linearly dependent. (5 marks)