**MACHAKOS UNIVERSITY**

**ISO 9001:2008 Certified**

**UNIVERSITY SUPPLEMENTARY EXAMINATIONS**

**SMA 407: MEASURE THEORY**

**(a) Answer ALL the questions in Section A and ANY TWO Questions in Section B**

**SECTION A**

**QUESTION ONE 30 MARKS**

a) Show that if E is Lebesgue measurable subset of$\mathbb{R}$ then $E^c$ is Lebesgue measurable subset (5 marks)

b) Prove that if $\mu$ is a measure defined on a $\sigma$ −algebra $\mathfrak{X}$. Then $\mu$ is monotonic, that is if $T \subset S$, then $\mu(T) \leq \mu(S)$. Furthermore if $\mu(T) \leq \infty$, then $\mu(S - T) = \mu(S) - \mu(T)$.
. (5 marks)

c) Show that if $\mu^*(F) = 0$, then F is L-measurable. (5 marks)

d) Prove that the space $(\mathbb{R}, \mathcal{M}, \mu)$, where $\mu$ is the lebesque measure is complete. (5 marks)

e) Prove that if $f, g: \rightarrow \mathbb{R}$ are two $\mathfrak{X}$ −measurable functions and $c$ be a real number then the function $cf$ is $\mathfrak{X}$ −measurable (5 marks)

f) Let $(f_n)$ be a sequence of $\mathfrak{X}$ −measurable function $f_n: X \rightarrow R_e$. Show that the set of points over which $(f_n)$ converges point wise is measurable. (5 marks)

**SECTION B**

**QUESTION TWO 20 MARKS**

a) Prove that if $f, g\colon \rightarrow \mathbb{R}$ are two $\mathfrak{X}$ −measurable functions and $c$ be a real number then the function

   i) $c + f$

   ii) $fg$ are $\mathfrak{X}$ −measurable. (@5 marks)

b) Let $(f_n)$ be a sequence of $\mathfrak{X}$ −measurable functions $f_n\colon X \rightarrow \mathbb{R}$ , then the functions $f_1 \cup f_2 \cup f_{3\cup} \ldots \ldots \ldots \cup f_n$ and $f_1 \cap f_2 \cap f_3 \cap \ldots \ldots \ldots \cap f_n$ is $\mathfrak{X}$ −measurable. (@5 Mrk)

**QUESTION THREE 20 MARKS**

a) Prove that $\mu^*(\{y\}) = 0$ for all $x \in \mathbb{R}$ (4 marks)

b) Prove that if $A \complement B$ then $\mu^*(A) \leq \mu^*(B)$, $A, B \in \mathbb{R}$ (5 marks)

c) Let $A = \{x \in R\colon -3 < x < 4\} \cup \{x \in R\colon 1 \leq x \leq 6\}$. Find $\mu^*(A)$ (4 marks)

d) Prove that lebesque outer measure $\mu^*$ is countably sub additive. (7 marks)

**QUESTION FOUR 20 MARKS**

a) Prove that if $f$ and $g$ both belong to $M^+$ and $f \leq g$, then $\int f du \leq \int g du$ (6 marks)

b) Prove that if $f \in M^+$ and if $E, F \in \mathfrak{X}$, with $E \complement F$ then $\int_E f du \leq \int_F f du$ (7 marks)

c) State and prove Fatous lemma. (7 marks)

**QUESTION FIVE 20 MARKS**

a) State and prove monotone convergence theorem. (10 marks)

b) Let $(X, \mathfrak{X})$ be a measurable space. Then if $f, g \in M^+(X, \mathfrak{X})$, then

   $f + g \in M^+(X, \mathfrak{X})$ (6 marks)

c) Prove that if $\varphi$ and $\rho$ are simple functions in $M^+(X, \mathfrak{X})$ and $c \geq 0$, then $\int c\varphi du = c \int \varphi du$ (4 marks)