



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS

SMA 360: PROBABILITY AND STATISTICS IV

DATE: 21/1/2021

TIME: 2.00-4.00 PM

INSTRUCTIONS TO CANDIDATES

Answer question one and any other two questions

QUESTION ONE

- a) Define the following terms giving illustrations;
- A random vector (2 marks)
 - A random matrix (2 marks)
- b) Suppose $\underline{X} = (x_1, x_2, \dots, x_p)^T$ and that $\underline{a}$ is a $P \times 1$ vector of constants. Show that $\text{Var}(\underline{a}^T \underline{X}) = \underline{a}^T \underline{\Sigma} \underline{a}$ where $\underline{\Sigma}$ is a variance covariance matrix. (4 marks)
- c) Suppose $\underline{\Sigma} = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$ Determine the correlation matrix, $\underline{\rho}$. (4 marks)
- d) Suppose $X \sim N(\mu, \sigma)$. Find the characteristic function of $Z = (x - \mu)/\sigma$. (5 marks)
- e) Let $\underline{X} \sim N_p(\underline{\mu}, \underline{\Sigma})$. Suppose $\underline{Y} = \underline{A}\underline{X} + \underline{b}$ where $\underline{A}$ is a $q \times p$ matrix of constants and $\underline{b}$ is a non zero $q \times 1$ vector of constants. Determine the distribution of $\underline{Y}$ by use of characteristic function. (5 marks)
- f) Compute the mean vector from the data matrix $\underline{X} = \begin{bmatrix} 4 & 1 \\ -1 & 3 \\ 3 & 5 \end{bmatrix}$ (4 marks)

- g) Show that $\Sigma = E(\underline{X}\underline{X}^T) - \underline{\mu}\underline{\mu}^T$ (4 marks)

QUESTION TWO

- a) Given two random variables X and Y with a joint probability function $P_{xy}(x,y)$ represented below;

		Y	
		0	1
X	-1	0.24	0.06
	0	0.16	0.14
	1	0.40	0.0

Find the variance covariance matrix, Σ . (10 marks)

- b) Suppose $\underline{X} = (X_1, X_2, X_3)^T$ with $\underline{\mu} = (2, 5, 3)^T$ and $\Sigma = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$. Find;

- Distribution of (x_1, x_2) .
- Distribution of $(x_1, x_3/x_2=2)$
- $R_{2,13}$

(10 marks)

QUESTION THREE

- a) Given that $f(x_1, x_2)$ is a bivariate normal density function. Show that $f(x_1/x_2)$ is $N\{\mu_1 + \frac{\sigma_{12}}{\sigma_{22}}(x_2 - \mu_2), \sigma_{11}(1 - \rho_{12}^2)\}$. (10 marks)

- b) Given that $f(x_1, x_2) = \alpha e^{-\frac{1}{2}\mathbf{x}^T \mathbf{A} \mathbf{x}}$ where α is some constant and

$$\mathbf{X} = X_1^2 + 2X_2^2 - X_1X_2 - 3X_1 - 2X_2. \text{ Identify } \underline{\mu}, \Sigma \text{ and hence express } \alpha \text{ in terms of } \pi.$$

(10 marks)

QUESTION FOUR

- a) Let X_1, X_2, X_3 , and X_4 be independent and identically distributed 3 X 1 random vectors with

$$\underline{\mu} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \text{ and } \Sigma = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}. \text{ Consider two linear combinations of the random vectors}$$

$$\frac{1}{2}X_1 + \frac{1}{2}X_2 + \frac{1}{2}X_3 + \frac{1}{2}X_4 \text{ and } X_1 + X_2 + X_3 - 3X_4. \text{ Find;}$$

- The mean vector for each linear combination of vectors

- ii. The covariance matrix for each linear combination of vectors and
 iii. The covariance between the two linear combination of vectors (10 marks)
- b) Given that $\mathbf{D} = \text{diagonal}(\sigma_1, \sigma_2, \dots, \sigma_p)$. Show that $\Sigma = \mathbf{D} \boldsymbol{\rho} \mathbf{D}$ where Σ is a variance covariance matrix and $\boldsymbol{\rho}$ is a correlation matrix. (4 marks)
- c) Suppose $\underline{X} \sim N_3(\underline{\mu}, \Sigma)$ and given that $Y_1 = X_1 - X_2$ and $Y_2 = X_1 - X_3$. Find
- i. $E(Y)$ (2 marks)
- ii. $\text{Var}(Y)$ (4 marks)

QUESTION FIVE

- a) Suppose $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ and $Q = (\underline{x} - \underline{\mu})^T \Sigma^{-1} (\underline{x} - \underline{\mu})$. Show that $Q \sim \chi_p^2$ (5 marks)
- b) Determine the $E(\mathbf{Q})$ given that $\mathbf{Q} = X_1^2 + 2X_2^2 + 3X_3^2 + 3X_1X_2 + 4X_1X_3$, $\underline{\mu} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\Sigma = \begin{bmatrix} 4 & 3 & 2 \\ 3 & 2 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (5 marks)
- c) Suppose $Y = X_1 + 2X_2 + X_3$ and $Z = X_1 - X_2 + 3X_3$. Determine the coefficient of correlation between Y and Z given that $\text{Var}(\underline{X}) = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}$. (5 marks)
- d) Let $\underline{Y} = (Y_1, Y_2, Y_3)$ having $\Sigma = \begin{bmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 2 \end{bmatrix}$. Suppose that $X = Y_1 - 2Y_2 + Y_3$. Compute $\text{Var}(X)$. (5 marks)