

MACHAKOS UNIVERSITY

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR, SECOND SEMESTER EXAMINATIONS

For Diploma in Mechanical Engineering. (TVET)

Date

Time **3 hours**

Instructions

The paper consists of **EIGHT** questions. Answer any **FIVE** questions.

ALL questions carry equal marks.

Show all your working

1. (a) Simplify the expressions;

i)
$$\frac{(1-x)^{\frac{1}{2}} - x(1-x)^{-\frac{1}{2}}}{1-x}$$

ii)
$$\frac{\log 729 - 4\log 3 + 2\log 27}{\log 243 - \log 27 + \log 9}$$
 without using logarithm tables (7marks)

- b) Solve the equations;

i) $\log_2 x + 2\log_4(x+1) = 1$

ii) $4^x = 2 + 16^{\frac{x}{4}}$ (13 marks)

2. a) Determine the values of p, q, and r such that $4x^2 - 3x + 12 = p(x + q)^2 + r$ (5 marks)

- b) The roots of the equation $ax^2 + bx + c = 0$ are α and $\alpha + 2$. Prove that

$b^2 = 4(a^2 + ac)$. (7marks)

- c) Solve the following simultaneous equations

$x + 2y - z = 1$

$x + 3y - 2z = 0$

$x + y + z = 4$

Use the method of substitution to solve the equation (8 marks)

3) a) Simplify the expression $5 \times 4^{3n+1} - 20 \times 8^{2n}$ (4 marks)

b) Find the values of:

i) $\frac{\log 15625}{\log 25} - 2$

ii) $\frac{8^{\frac{2}{3}} + 4^{\frac{3}{2}}}{16^{\frac{3}{4}}}$ (6 marks)

c) Given that $2\log 8N = p$, $\log 22N = q$ and that $q - p = 4$, determine the value of N.

(10 marks)

4.a) Given that $\sin A = \frac{12}{13}$ and $\cos B = \frac{4}{5}$ where A is obtuse and B is acute, determine the values of ;

i) $\sin (A - B)$

ii) $\tan (A + B)$ (5 marks)

b) Prove the identities:

i) $\frac{1 - \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 - \cos \theta} = 2 \operatorname{cosec} \theta$

ii) $\tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$ (8 marks)

c) Given $t = \tan 22 \frac{1}{2}$

i) Show that $\tan 45^\circ = \frac{2t}{1 - t^2}$;

ii) Hence solve the equation:

$t^2 + 2t - 1 = 0$, leaving your answer in surd form. (7 marks)

5.a) Express in polar co-ordinates the position :

i) $P_1 (3 \ 4)$ ii) $P_2 (-5 \ -8)$ (6 marks)

b) obtain the Cartesian equations of;

i) $r = 5(1 + 2 \cos \theta)$

ii) $r = a \tan \theta$ (7 marks)

c) Find the cartesian equations of the loci;

i) $x = t^2 + 4$ and $y = t - 3$ Type equation here.

ii) $x = 5\cos\theta$ and $y = 4\sin\theta$ (7 marks)

6.a) Obtain the (i) polar equation of the of the loci $x^2 + y^2 - 2x = 0$

(ii) parametric equation of the locus $x^3 + y^3 = 3xy$ (7 marks)

b) Express $\frac{6-7j}{j(5-2j)} + \frac{3}{2j}$ (5 marks)

c) Solve for $0^\circ \leq x \leq 360^\circ$ given;

$$12\sin 2x + 12\cos x - 6\sin x - 3 = 0 \quad (8 \text{ marks})$$

7.a) Given the complex numbers $Z_1 = 4 + 3j$, $Z_2 = 1 + j$ and $Z_3 = 1 - 2j$, express

$$Z = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3} \quad \text{in the form } a + bj. \quad (8 \text{ marks})$$

b) Use De Moivre's theorem to prove that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$ (5 marks)

c) Given that $Z = j$ is one root of the equation $Z^4 - 2Z^3 + 3Z^2 - 2Z + 2 = 0$,
determine the other roots. (7 marks)

8.a) solve the following equations for all values of θ between 0° and 360° .

$$2\sin\theta - 3\cos\theta = 2 \quad (7 \text{ marks})$$

b) Solve for x in the following equations

$$\text{i) } \log_3(2x-3) = -1 \quad (3 \text{ marks})$$

$$\text{ii) } 3^{2x} = 4(3^x) + 3 \quad (4 \text{ marks})$$

c) Show that $\frac{1+\tan^2 B}{1+\cot^2 B} = \tan^2 B$ (6 marks)