MACHAKOS UNIVERSITY

University Examinations for 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR FIRST SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (INFORMATION TECHNOLOGY)

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

BACHELOR OF EDUCATION (SPECIAL NEEDS)

BACHELOR OF ECONOMICS AND STATISTICS

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF EDUCATION (ARTS)

BACHELOR OF ECONOMICS

SMA 202: LINEAR ALGEBRA I

DATE: 17/8/2021 **TIME: 8:30 – 10:30 AM**

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Evaluate $\begin{vmatrix} 3 & 2 & 5 \\ 4 & 7 & 9 \\ 1 & -1 & 3 \end{vmatrix}$ (4 marks)

b) Given the system

$$x + y + z = 6$$
$$3x + 4z + 3y = 20$$
$$2x + y + 3z = 13$$

Solve the system using reduction to echelon form method (4 marks)

c) Determine the acute angle between the lines;

$$\frac{x-1}{5}=\frac{y+2}{4}=\frac{z}{6} \qquad and \qquad \frac{x+3}{2}=\frac{y-5}{7}=\frac{z+1}{-1}$$ (4 marks)

d) Calculate the equation of the plane through the point $(2, 1, -3)$ given that it is perpendicular to the vector $5i+6j+7k$ (4 marks)

e) Given the matrix $A=\begin{pmatrix} -4 & 1 & -6 \\ 1 & 2 & -5 \\ 6 & 3 & -4 \end{pmatrix}$. Evaluate A^{-1} (4 marks)

f) Show that the vectors

$u=(6,\ 2,\ 3,\ 4)$, $v=(0,\ 5,\ -3,\ 1)$, *and* $w=(0,\ 0,\ 7,\ -2)$ are independent.

(4 marks)

g) Let v be the vector space of all functions from the real field $\Re$ into $\Re$. Show that w is a subspace of v given that;

i.) $w=\{f : f(3)=0\}$ i.e. w consists of those functions which map 3 into 0. (3 marks)

ii.) $w=\{f : f(7)=f(1)\}$ i.e. w consists of those functions which assign the same value to 7 and 1. (3 marks)

QUESTION TWO (20 MARKS)

a) Solve the following system of equations using the Crammer's rule (8 marks)

$$-y-2z=-8$$
$$x+3z=2$$
$$7x+y+z=0$$

b) Determine the value of m such that the vectors $\vec{a}=2i+j+4k, and\ \vec{b}=3i+2j+mk$ are coplanar. (5 marks)

c) Given that;

$$\vec{a}=a_1i+a_2j+a_3k$$
$$\vec{b}=b_1i+b_2j+b_3k$$
$$\vec{c}=c_1i+c_2j+c_3k$$

Show that $\vec{a}\cdot(\vec{b}\times\vec{c})=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ (7 marks)

QUESTION THREE (20 MARKS)

a) Show that the line $y = x + 1$ is not a subspace of $\Re$ i.e
$W = \{(x, y)/ y = x + 1\} = \{(x, x + 1)/ x \in \Re\}$ (5 marks)

b) Calculate the area of a triangle having vertices at
$P(1 , 3, 2) , Q(2 , -1 , 1) \text{ and } R(-1 , 2 , 3)$ (5 marks)

c) Determine the value of h such that $(1 , 3, 3) , (-2 , -6 , h) \text{ and } (3 , 9 , -1)$ are Linearly independent (5 marks)

d) Calculate the work done in moving an object a long a vector $\vec{A} = 3i + 2j - 5k$ if the applied force is $\vec{F} = 2i - j - k$ (5 marks)

QUESTION FOUR (20 MARKS)

a) Solve the system of linear equations below by matrix inversion method. (8 marks)

$$x - 8y + z = 4$$
$$-x + 2y + z = 2$$
$$-y + x + 2z = -1$$

b) Use Gram-Schmidt process to find an orthogonal basis spanning the same subspace of R^n as the vectors $< 0,2,1,-1 >$, $< 0,-1,1,6 >$ and $< 0,2,2,3 >$ (7 marks)

c) Find the basis and dimension of the set spanned by the vectors $(1,-1,2)$, $(0,5,-8)$, $(3,2,-2)$, and $(8,2,0)$. (5 marks)

QUESTION FOUR (20 MARKS)

a) Solve the following system using Crammers method

$$2x - 5y + 2z = 7$$
$$x + 2y - 4z = 3$$
$$3x - 4y - 6z = 5$$

(7 marks)

b) Consider the system

$$10x_1 + 7x_2 + 8x_3 + 7x_4 = 32$$
$$5x_2 + 7x_1 + 6x_3 + 5x_4 = 23$$
$$8x_1 + 9x_4 + 6x_2 + 10x_3 = 33$$
$$10x_4 + 5x_2 + 7x_1 + 9x_3 = 31$$

Solve using reduction to echelon form method (13 marks)