# MACHAKOS UNIVERSITY

**University Examinations 2018/2019**

**SCHOOL OF PURE AND APPLIED SCIENCES**

**DEPARTMENT OF PHYSICAL SCIENCES**

**THIRD YEAR SUPPLEMENTARY EXAMINATION FOR**

**BACHELOR OF EDUCATION (SCIENCE)**

**SPH 301: QUANTUM MECHANICS 1**

**DATE: 26/7/2019** **TIME: 8:30 – 10:30 AM**

## INSTRUCTIONS:

a) The paper consists of **two** sections.
b) Section **A** is **compulsory** (30 marks).
c) Answer any **two** questions from section **B** (each 20 marks).

## QUESTION ONE (COMPULSORY) (30 MARKS)

a) Explain three failures of classical mechanics from the photoelectric experiment that led to quantum mechanics approach (6 marks)
b) Write the Laplacian operator in mathematical form (2 marks)
c) Show that for a one-dimensional square - integrable wave-packet,

$$\int_{-\infty}^{\infty} j(x)dx = \frac{\langle p \rangle}{m}$$

Where$j(x)$, is the probability current? (7 marks)

d) State the expectation value for position, momentum and energy for a normalised wave function (3mks)
e) Show that $[x, p_x] = i\hbar$ (5 marks)

f) Write down expression for time independent Schrodinger equation in one dimension. (2 marks)

g) A particle of mass $m$ is trapped in a one dimensional box with a potential described by

i. $V(x) = \begin{cases} 0 & 0 \leq x \leq a \\ \infty & \text{Otherwise} \end{cases}$

Solve the Schrödinger equation for this potential. Taking $\Psi(x,t) = \Psi(x)e^{-iEt/\hbar}$ (8 marks)

**QUESTION TWO (20 MARKS)**

a) State two properties of Schrödinger equation (2mks)

b) Derive Time – dependent Schrödinger equation and hence write it in three dimension given that the wave equation is $\Psi = Ae^{i(kx-\omega t+\Phi)}$ (18 marks)

**QUESTION THREE (20 MARKS)**

a) What are the two conditions for normalization of a wave function (2 marks)

b) Consider a particle described by a wave function *ρ(r, t)*. Calculate the time-derivative $\frac{\partial\rho(r,t)}{\partial t}$ where ρ*(r, t)* is the probability density and show that the continuity equation $\frac{\partial\rho(r,t)}{\partial t} + \nabla.\vec{\mathrm{J}}(\mathrm{r},\mathrm{t}) = 0$ is valid, where ***J****(r,t)* is the probability current (18 marks)

**QUESTION FOUR (20 MARKS)**

a) State the mathematical equation operators for momentum, kinetic energy and hamilitonian as used in quantum mechanics (3 marks)

b) Show that the momentum operator $-i\hbar\frac{\partial}{\partial x}$ is Hermitian operator. Obtain eigenfunction of momentum operator (17 marks)

**QUESTION FIVE**

a) The wavelength and frequency in a waveguide are related by $\lambda = \frac{c}{\sqrt{v^2 - v_0^2}}$ Express the group velocity $V_g$ in terms of $c$ and phase velocity $V_P = v\lambda$ (8 marks)

b) Consider a particle of mass $m$ held in a one-dimensional potential $V(x)$. Suppose that in some region $V(x)$is constant $V(x) = V$. For this region, find the stationary states of the particle when (a) $E > V$,(b) $E < V$, and (c) $E = V$, where E is the energy of the particle. (12 marks)