



MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION

SMA 334: FLUID MECHANICS II

DATE: 29/3/2021

TIME: 11.00-1.00 PM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (COMPULSORY – 30 MARKS)

- (a) Define the following terms as used in fluid mechanics
- (i) A stream line (1 mark)
 - (ii) A source (1 mark)
 - (iii) Incompressible fluid (1 mark)
- (b) Determine the equation of a straight line passing through the point (a, 0) if the two dimensional flow is described by $u = \frac{-y}{b^2}$ and $v = \frac{x}{a^2}$. (6 marks)
- (c) Given that $\phi = 2xy$ determine ψ . (5 marks)
- (d) Find the stream function of the two dimensional motion due to the equal sources and an equal sink midway between them. Sketch the streamlines (6 marks)
- (e) Find the image of $|z - 2i| = 2$ under the mapping $w = \frac{1}{z}$ (5 marks)
- (f) If ϕ and ψ are functions satisfying Laplace equation, show that $S + it$ is analytic where $S = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}$ at $t = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}$. (5 marks)

QUESTION TWO (20 MARKS)

- (a) Prove that $\psi = \log[(x - 1)^2 + (y - 2)^2]$ is harmonic in every region which does not include the point (1,2). Find a function ϕ such that $\phi + i\psi$ is an analytic function of the complex variable $z = x + iy$. Express $\phi + i\psi$ as a function of z . (10 marks)
- (b) Show that the following factor represents possible flow phenomena of irrotational type. $\phi = u(x) \frac{\theta}{r}$ (10 marks)

QUESTION THREE (20 MARKS)

- (a) A viscous fluid is confined on one side of an infinite porous plane. If the velocity of the suction v_0 at the plane is constant and also the plane moving parallel to itself with constant velocity u , find the velocity distribution and calculate the drag at the plane. (10 marks)
- (b) State and derive the Blasius theorem. (10 marks)

QUESTION FOUR (20 MARKS)

- (a) (i) State THREE types of standard transformations. (3 marks)
- (ii) For the conformal transformation $w = z^{\frac{1}{2}}$, show that the circle $|z - 1| = 1$ transforms into $\rho^2 = 2 \cos 2\phi$, where $w = \rho e^{i\theta}$ in the w -plane. (7 marks)
- (b) Discuss in brief the following
- (i) Lubrication theory (5 marks)
- (ii) Role of Reynold's number in boundary layer (5 marks)

QUESTION FIVE (20 MARKS)

- (a) Derive the Euler's equation
- $$\rho \frac{\partial \vec{v}}{\partial t} + \rho \vec{v} \cdot \nabla \vec{v} = -\nabla p$$
- Where ρ , $\vec{v}$ and p are density, velocity and pressure respectively. (10 marks)
- (b) A fluid is flowing steadily between two fixed parallel plates under a constant pressure gradient $p = \left(\frac{-\partial p}{\partial x}\right)$. Show that the velocity distribution is $u = \frac{p}{2\mu}(d^2 - y^2)$ where $2d$ is the distance between the plates and y is measured from the mid-point. (10 marks)