



MACHAKOS UNIVERSITY
University Examinations 2021/2022

**FOURTH YEAR SUPPLEMENTARY/SPECIAL EXAMINATION FOR
BACHELOR OF EDUCATION SCIENCE, EDUCATION ARTS, MATHEMATICS AND
MATHEMATICS & COMPUTER SCIENCE**

SMA 401: TOPOLOGY II

DATE: 6/3/2023

TIME: 2.00 – 4.00 PM

INSTRUCTIONS TO CANDIDATES

Answer ALL the questions in Section A and ANY TWO Questions in Section B

SECTION A

QUESTION ONE (30 MARKS) COMPULSORY

- a) Define the following terms
 - i) Finite intersection property
 - ii) Sequentially compact
 - iii) Compact spaces
 - iv) Separated sets
 - v) Locally compact sets (5 marks)
- b) Prove that the open interval $A = (0, \frac{1}{2})$ on the real line $\mathbb{R}$ with the usual topology is not compact. (5 marks)
- c) Consider the following topology on $X = \{a, b, c, d, e\}$ $\rho = \{X, \emptyset, \{a, b, c\}, \{c, d, e\}, \{c\}\}$. Now $A = \{a, d, e\}$. Show that A is disconnected. (5 marks)
- d) Prove that if X is sequentially compact then it is countably compact. (5 marks)
- e) Prove that a set is disconnected if and only if it is not a union of two non-empty separated sets. (5 marks)
- f) Prove that if X is compact then it is countably compact. (5 marks)

SECTION B

QUESTION TWO (20 MARKS)

- a) Prove that if A and B are disjoint compact subsets of Hausdorff spaces X . Then $\exists$ open set G and H such that $A \subset G$ and $B \subset H$ and $G \cap H = \emptyset$
(6 marks)
- b) Let $\mathbb{Z}$ be the set of integers. Is it sequentially compact. (3 marks)
- c) Let $G \cap H$ be a disconnection of A . show that $G \cap A$ and $H \cap A$ are separated. (5 marks)
- d) Let $G \cup H$ be a disconnection of A and B be a connected subset of A . Show that $B \cap G = \emptyset$ or $B \cap H = \emptyset$. (6 marks)

QUESTION THREE (20 MARKS)

- a) Show that an infinite subset A of a discrete space X is not compact (7 marks)
- b) Prove that a closed subset F of a compact set X is also compact. (7 marks)
- c) Prove that a topological space X is compact if and only if $\{F_i\}$ of closed subsets of X satisfies the finite intersection property. (6 marks)

QUESTION FOUR (20 MARKS)

- a) Define hereditary as used in topological spaces (2 marks)
- b) T_0 and T_1 spaces are hereditary prove (6 marks)
- c) Let (X, ρ) be a topological space (X, ρ) is a T_1 space iff each singleton subset $\{x\}$ is closed in (X, ρ) (6 marks)
- d) Prove that if (X, ρ) is a topological space which is a T_2 . Then every convergent sequence of points of X has a unique limit. (6 marks)

QUESTION FIVE (20 MARKS)

- a) Prove that every compact subset of a Hausdorff space is closed. (5 marks)
- b) Show that if A and B are non-empty separated sets. Then, $A \cup B$ is disconnected (5 marks)
- c) Show that every metric space is Hausdorff space (5 marks)
- d) Prove that if F is closed subset of a compact space X , then F is also compact. (5 marks)