MACHAKOS UNIVERSITY COLLEGE

(A Constituent College of Kenyatta University)
University Examinations for 2015/2016 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST SEMESTER EXAMINATION FOR DEGREE IN
BACHELOR OF SCIENCE IN MATHEMATICS AND STATISTICS
BACHELOR OF EDUCATION (SCIENCE)
BACHELOR OF EDUCATION (ARTS)

SMA 300: REAL ANALYSIS 1

Date: 2/8/2016 **Time: 2:00 – 4:00 PM**

INSTRUCTION:

Answer Question One and any other Two Questions

QUESTION ONE: 30 MARKS (COMPULSORY)

a) Given the following typical sets;

N : The set of Natural numbers

Z : The set of integers

Q : The set of Rational Numbers

Q^c : The set of Irrational Numbers

R : The set of Real numbers

i) Starting with Real numbers construct a chart/ diagram to depict the relationships of set inclusion among the above sets. (3 marks)

ii) Identify with reasons which of the above sets have a field structure. (2 marks)

b) Identify the four properties that must be satisfied by a filed in order to possess the order structure. (4 marks)

c) State the completeness axiom for R, the set of real numbers. (2 marks)

d) Show that $\sqrt{2}$ is irrational. (4 marks)

e) State the Supremum and Infimum of the following sets;

$S_1 : \left\{\frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \dots\right\}$ (2 marks)

$S_2 : \left\{\frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \dots\right\}$ (2 marks)

f) By computing limsup(x_n) and lim inf(x_n) determine the convergence or divergence of the sequence.

$x_n = (-1)^n \left[4 + \frac{5}{n}\right] \ \forall n \in N.$ (4 marks)

g) For all real numbers x and y prove the triangular inequality

$|x + y| \leq |x| + |y|$ (4 marks)

h) Show that the square of an odd integer is odd. (3 marks)

QUESTION TWO: (20 MARKS)

a) Give a general definition of neigbourhood of a point in R (the set of real numbers). Using relevant examples. (3 marks)

b) Let P and T be neighbourhoods of a point y. Prove that the intersection $P \cap T$ is also a neighbourhood of y. (7 marks)

c) Using relevant examples define a limit point of a set. (3 marks)

d) Show with reasons that the set:

$\left\{2, -2, 2\frac{1}{2}, -2\frac{1}{2}, 2\frac{1}{3}, -2\frac{1}{3} \dots\right\}$

is closed but not open. (7 marks)

QUESTION THREE: (20 MARKS)

a) Give the definition of an irrational number. Give examples. (3 marks)

b) Show that $\sqrt{m+1} - \sqrt{m-1}$ is irrational (7 marks)

c) Does the equation $x^2 + 1 = 0$ have a solution in R. Show your workings clearly. (3 marks)

d) i) Give the definition of an open set, and identify 2 examples. (3 marks)

ii) Show that every open interval in R is an open set. (5 marks)

QUESTION FOUR (20 MARKS)

a) Distinguish clearly between a sequence and a series. (4 marks)

b) Identify the criterion for convergence of a sequence hence show that the sequence

$$(x_n) = 3 + \frac{(-1)^n}{n^2}$$

converges to 3 from first principles (by definition). (5 marks)

c) Use the Ratio test to determine the convergence or divergence of $\sum_{n=0}^{\infty} \frac{2^n}{n!}$ (5 marks)

d) Calculate the following limits

i) $\lim_{x\to\infty} \frac{2x^2+5}{3x^2+1}$ (3 marks)

ii) $\lim_{x\to -1} \frac{(x+2)(3x-1)}{x^2+3x-2}$ (3 marks)

QUESTION FIVE: (20 MARKS)

a) Let A and B be non- empty subsets of R and define the set

$$A + B = \{x + y : x \in A, y \in B\}.$$

Show that

i) If A and B are bounded above then so is $A + B$ and Sup($A + B$)= Sup(A) + Sup (B). (5 marks)

ii) If A and B are bounded below then so $A + B$ and Inf ($A + B$) = Inf A+ Inf B. (5 marks)

b) Use comparison test to show that the series $1 + \frac{1}{2!} + \frac{1}{3!} + \ldots$ is convergent. (5 marks)

c) Let x and y be positive real numbers, show that $x < y$ iff $x^2 < y^2$. (5 marks)