



MACHAKOS UNIVERSITY

University Examinations for 2021/2022

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (ACTUARIAL SCIENCE)

SAC 301: ACTUARIAL MATHEMATICS II

DATE: 18/3/2022

TIME: 8.30-10.30 AM

INSTRUCTIONS: Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY) 30 MARKS

a) Define the following actuarial notations

i) q_{xy} (2 marks)

ii) $T_{\overline{xy}}$ (2 marks)

b) You are given the following extract from a double decrement table.

Age (x)	$(al)_x$	$(ad)_x^d$	$(ad)_x^w$
50	100000	175	2490
51	97335	180	2160
52	94995		

Table 1

Where d and w refer to death and withdrawal respectively

Calculate

i) $(ap)_{51}$ (2marks)

ii) ${}_2(aq)_{50}^w$ (4 marks)

c) Calculate ${}_3q_{\overline{50:50}}$ using AM92 ultimate. (5 marks)

- d) In a combined sickness and mortality table

$$K_{x+1} = 554,405$$

$$z_x = 1.129$$

$$D_x = 24,510$$

$$D_{x+1} = 23,425$$

Estimate K_x . (5 marks)

- e) A certain life office sells assurance policies with term 3 years to lives aged 70. For each policy, the profit vector is estimated to be $(-50, 30, 30)$. Given that the mortality of the policyholder is expected to follow AM92 ultimate, calculate the profit signature per policy sold. (5 marks)
- f) Consider a person now aged exactly 25 whose annual salary rate is currently £9,192. Estimate his annual salary rate at exact age 53 (5 marks)

QUESTION TWO (20 MARKS)

A life insurance company issues a number of 3-year term assurance contracts to lives aged exactly 60. The sum assured under each contract is £200,000, payable at the end of the year of death. Premiums are payable annually in advance for the term of the policy, ceasing on earlier death.

The company carries out profit tests for these contracts using the following assumptions:

- Initial expenses: £200 plus 35% of the first year's premium
 - Renewal expenses: £25 plus 3% of the annual premium, incurred at the beginning of the second and subsequent years
 - Mortality: AM92 Ultimate
 - Investment return: 7% per annum
 - Risk discount rate: 15% per annum
 - Reserves: One year's office premium
- i) Show that the office premium, to the nearest pound, is £2,527, if the net present value of the profit is 25% of the office premium. (13 marks)
- ii) Calculate the cash flows if the company held zero reserves throughout the contract, using the premium calculated in part (i). (5 marks)
- iii) Explain why the company might not hold reserves for this contract and the impact on profit if they did not hold any reserves. (2 marks)

QUESTION THREE (20 MARKS)

In a certain population, forces of decrement are assumed to be constant over individual years of age.

The following independent forces of decrement will be assumed for this population between the exact ages of 50 and 52.

Age (x)	Force of decrement for the year of age due to mortality	Force of decrement from exact age x due to sickness
50	0.011	0.075
51	0.012	0.081

Table 2

Construct a double decrement table including the two decrements of mortality and sickness, for this population between exact ages 50 and 52, assuming a radix of $(al)_{50} = 100,000$

QUESTION FOUR (20 MARKS)

- a) Define the following terms;
- Deferred period (2 marks)
 - The off period. (2 marks)
- b) The mean present value of a sickness benefit may be evaluated by direct evaluation of a sum, by approximate integration or by commutation functions. Derive the mean present value of a sickness benefit of £ 1 per week for life x expressed by commutation functions. (6 marks)
- c) On 1 January 1995 a friendly society had in force a large group of policies providing certain sickness and life assurance benefits. These policies had all been issued 10 years ago to lives now aged 45. The benefits cease when the policyholders reach age 65, and the policies have level weekly premiums payable throughout the term of the policy. Each policy provides an income of £20 per week during any period of sickness which has lasted more than 6 months, and a lump sum of £20,000 immediately on death before age 65. Premiums are waived during sickness (even when no benefit is payable), and the off-period is as in the Manchester Unity sickness tables. The basis for the calculation of premiums and reserves is:
- Mortality: E.L.T. 12 – Males.
 - Interest: 4% p.a.

- Sickness: Manchester Unity Experience 1893-97 (Occupation Group AHJ)
 - Expenses: 20% of all premiums, including those waived.
- i. Calculate the weekly premium for each of these policies. (5 marks)
 - ii. Calculate the reserve per policy in force on 1 January 1995. (5 marks)

QUESTION FIVE (20 MARKS)

- a) Proof that

$$P[K_{xy} = K] = {}_k|q_{xy} \quad (5 \text{ marks})$$

- b) i) Describe the benefits typically provided by a salary-related pension scheme for active members on age retirement. (5 marks)
- ii) A company pension scheme provides the following benefits for all members:
1. A pension on retirement (on grounds of ill-health or of age) of one-eightieth of final pensionable salary for each year of service (including fractions)
 2. A lump sum on retirement of 3 times the annual pension
 3. On death in service, a lump sum of £30,000
 4. On withdrawal from service, a return of the employee's contributions, accumulated at 3% per annum compound.

Final pensionable salary is defined as the average annual salary in the three years immediately before retirement. All members who reach age 65 retire immediately.

Employees contribute to the scheme at the rate of 3% of salary, payable continuously.

Salaries are revised continuously. The employer's contribution rate is assessed for each member separately, and is such that the prospective reserve for each new entrant is zero.

Expenses are ignored

- i. Derive a formula, in terms of suitable commutation functions, for valuing benefit (1) above in respect of a new entrant aged x with annual salary rate SAL. (You need not define the service table functions.) (4 marks)
- ii. In respect of a new entrant aged x with annual salary rate SAL, give formulae for valuing benefits (2), (3) and (4) above, using suitable commutation functions. (You need not derive the formulae.) (6 marks)