MACHAKOS UNIVERSITY

University Examinations 2020/2021 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

FIRST YEAR SPECIAL/SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

SST 102: DISCRETE MATHEMATICS

DATE: 12/8/2021 **TIME: 2.00-4.00 PM**

INSTRUCTIONS:

Answer Question One and any other two questions

QUESTION ONE - (30 MARKS)

a) Explain the meanings of the following terms

i. a *binary relation* on a set A, (2 marks)

ii. a *surjective function* from a set A to a set B, (2 marks)

iii. a *recurrence relation* (2 marks)

b) Let a, x, y be integers and suppose that $a \mid x$ and $a \mid y$. Prove that $a \mid (ux - vy)$ for any $u, v \in \mathbb{Z}$. (5 marks)

c) Consider the sets $A = \{3,45,201\}$ and $B = \{2,17,45,48,300\}$ and let R be a relation defined from A to B such that for any $a \in A$ and $b \in B$ then aRb if and only if a is greater than B or if a divides b. Determine the elements of R. (5 marks)

d) Let $g: \mathbb{R} \to \mathbb{R}$ be the function defined by $g(x) = x^2 + 6$. Determine whether g is injective and/or surjective. (5 marks)

e) Determine the number of non-negative integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 121$. (4 marks)

f) Four individuals A, B, C, D intends to register a political coalition agreement. One of them is to be the president, one the deputy president, one the leader of majority in the parliament and one is to be the speaker of the parliament. Determine the number of ways that the positions can be assigned? (5 marks)

QUESTION TWO (20 MARKS)

a) Explain the meaning of the terms

i. transitive binary operation on a set A, (2 marks)

ii. relatively prime numbers $a, b \in \mathbb{Z}$. (2 marks)

b) Define a relation $\sim$ on the set $\mathbb{Z}$ as follows: $a \sim b$ if and only if $a - b$ is divisible by 5.

i. Determine if $\sim$ is an equivalence relation. (4 marks)

ii. Find the equivalence class of 2. (3 marks)

c) Use the Euclidean algorithm

i. to find $d = \gcd(5613, 772)$, (4 marks)

ii. express $\gcd(5613, 772)$ in the form $d = 5613m + 772n$. (5 marks)

QUESTION THREE (20 MARKS)

a) Let a, b and c be integers. Prove that

i. If $a \mid b$ and $a \mid c$ then $a \mid (b + c)$. (2 marks)

ii. If $a \mid b$ and $a \mid bx$ for any integer x. (2 marks)

iii. If $a \mid b$ and $b \mid c$, the $a \mid c$. (2 marks)

b) Let a, b, m be integers with $m > 0$ and suppose that $a \equiv b (mod\ m)$.

i. Prove that $a + x \equiv b + x (mod\ m)$ for any integer x. (4 marks)

ii. Solve the equation $374x \equiv 51 (mod\ 289)$ (5 marks)

c) Each user on a computer system has a password which is six to eight characters long. Each character in a generated password can be a lowercase alphabetic letter or a numeric digit. Each password must contain at least one digit. Determine the possible number of unique passwords that can be generated.

(5 marks)

QUESTION FOUR (20 MARKS)

a) Let $\mathbb{Z}$ denote the set of integers. Consider the functions $f: \mathbb{Z} \to \mathbb{Z}$ and $g: \mathbb{Z} \to \mathbb{Z}$ given respectively by $f(n) = 2n + 3$ and $g(n) = 3n^2 + 2$. What is

i. the composition $f \circ g$, (2 marks)

ii. the composition $g \circ f$. (4 marks)

b) Suppose that there are 3 distinct courses in computational mathematics, 5 distinct courses in statistics and 8 distinct courses in fluid mechanics. Determine the number of distinct ways can a student choose 2 courses in computational mathematics, 3 courses in statistics and 4 courses in fluid mechanics. (5 marks)

c) Consider the sequence $\{a_n\}$ defined by $a_n = -a_{n-1} + 20a_{n-2}$. Solve this recurrence relation subject to the initial conditions $a_0 = 1, a_1 = 14$. (5 marks)

d) Suppose that n, k are positive integers such that $1 \leq k \leq n$. Prove that . (4 marks)

$$\binom{n}{k} = n\binom{n-1}{k-1}$$

QUESTION FIVE (20 MARKS)

a) Let $\sim$ be a binary relation defined on $\mathbb{R}^2$ by $(a, b) \sim (x, y)$ if and only if $(a, b) = \lambda(x, y)$ for some real number $\lambda \neq 0$.

   i. Prove that $\sim$ is an equivalence relation on $\mathbb{R}^2$. (5 marks)

   ii. Describe the equivalence class of $(1,1)$ under the equivalence relation $\sim$ defined above. (2 marks)

b) Prove that $\binom{n+1}{r} = \binom{n}{r-1} + \binom{n}{r}$. (5 marks)

c) Assume that a country currently with a population of 40 million people has a population growth of 3%.

   i. Determine how big its population will be after 15 have ellapsed. (4 marks)

   ii. If the country receives 200,000 new immigrants every year, determine its population in 15 years time assuming that all the immigrants arrive at the end of the year and reproduce at the same rate as the native population. (4 marks)