MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH YEAR SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF EDUCATION (SCIENCE)

SMA 432: PARTIAL DIFFERENTIAL EQUATION II

DATE: 25/7/2019 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

Attempt question ***one (compulsory***) and any other ***two questions***.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) Solve the differential equation $\frac{dx}{\cos(x+y)} = \frac{dy}{\sin(x+y)} = \frac{dz}{z}$ (5 marks)

b) Determine the orthogonal trajectories on the conicoid $(x+y)z = 1$ of the conics in which it is cut by the system of planes $x - y + z = k$, where k is a parameter. (5 marks)

c) Solve the Lagrange's equation $y^2 p - xyq = x(z-2y)$ (5 marks)

d) Work out the characteristics and the corresponding transport equations of the system

$$xyu_x - v_y = 0$$
$$xu_y - v_x = 0$$
(5 marks)

e) Determine the complete integral of $z = px + qy + p^2 + q^2$ using Charpit's method (5 marks)

f) Eliminate the arbitrary function f from the equation $f\left(x^2 + y^2 + z^2, z^2 - 2xy\right) = 0$ (5 marks)

QUESTION TWO (20 MARKS)

a) Using Jacobi's method determine a complete integral of $p_1 x_1 + p_2 x_2 = p_3^2$ (10 marks)

b) Consider the partial differential equation $p^2 + q^2 - 2pq\tanh 2y = \sec h^2 2y$;

   i) Write its Charpit's auxiliary equations (2 marks)

   ii) Solve the p.d.e by considering its Charpit's auxiliary equations. (8 marks)

QUESTION THREE (20 MARKS)

Solve ;

i) $\dfrac{dx}{x^2} = \dfrac{dy}{y^2} = \dfrac{dz}{nxy}$ (7 marks)

ii) $\dfrac{dx}{y^2(x-y)} = \dfrac{dy}{-x^2(x-y)} = \dfrac{dz}{z(x^2-y^2)}$ (8 marks)

iii) Show that $(2x + y^2 + 2xz)dx + 2xydy + x^2dz = 0$ is integrable (5 marks)

QUESTION FOUR (20 MARKS)

a) Determine the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to $z = c$

(8 marks)

b) Solve the semi-linear equation $x^2 \dfrac{\partial z}{\partial x} + y^2 \dfrac{\partial z}{\partial y} = (x+y)z$ (7 marks)

c) Consider the partial differential equation $z = px + qy + p + q - pq$ with a complete integral of the form $z = ax + by + p + a + b - ab$; where a *and* b are arbitrary constants. Determine a general solution by finding the envelope of those planes that pass through the origin. (5 marks)

QUESTION FIVE (20MKS)

a) Calculate the integral surface of the quasi-linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2 z)$ which contains the straight line $x + y = 0$, $z = 1$

(8 marks)

b) in the one-dimensional unsteady flow of compressible fluid, the velocity u *and* c where $c^2 = \dfrac{dp}{d\rho}$, p is pressure and ρ is density satisfying the following equations.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + 2c\frac{\partial c}{\partial x} = 0$$

$$2\frac{\partial c}{\partial t} + 2u\frac{\partial c}{\partial x} + c\frac{\partial u}{\partial x} = 0$$

Prove that the characteristics are given by the differential equation $dx = (u \pm c)dt$ and that on the characteristic $dx = (u + c)dt$, $u + 2c$ is a constant. (12 marks)