



MACHAKOS UNIVERSITY

University Examinations 2019/2020 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR SPECIAL /SUPPLEMENTARY EXAMINATION FOR

BACHELOR OF SCIENCE IN STATISTICS AND PROGRAMMING

SMA 336: ORDINARY DIFFERENTIAL EQUATION II

DATE: 19/01/2021

TIME: 8.30-10.30 AM

INSTRUCTIONS TO CANDIDATES

Attempt question *one (compulsory)* and any other *two questions*.

QUESTION ONE (30 MARKS)

a) Solve the initial value problem $\frac{d^2 y}{dx^2} + y = 0$; $y(0) = 3$, $y'(0) = -4$ (5 marks)

b) Consider the initial value problem

$$\frac{d^3 z}{dt^3} - 3\frac{d^2 z}{dt^2} + 2\frac{dz}{dt} - z = \sin t \quad z(0) = 1, z'(0) = 0, z''(0) = 3$$

Reduce the equation to a system of first order linear differential equations. (5 marks)

c) Consider the initial – value problem $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 3$. Determine whether the differential equation has a unique solution (5 marks)

d) Solve the system of differential equations below using the matrix method

$$\begin{aligned} \frac{dx_1}{dt} &= 3x_1 - 2x_2 \\ \frac{dx_2}{dt} &= -x_1 + 2x_2 \end{aligned} \quad (5 \text{ marks})$$

e) Determine the integrability of the differential equation $zdx + zdy + 2(x + y + \sin z)dz = 0$ (5 marks)

- f) Consider the differential equation $x^2(1-x)y'' + (1-x)y' + y = 0$
- i. Determine the singular points (2 marks)
- ii. Determine the nature of the singular points in (i) above (3 marks)

QUESTION TWO (20 MARKS)

Use the method of Frobenius to solve the differential equation

$$2x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + (x-5)y = 0$$

QUESTION THREE (20 MARKS)

Consider the system of differential equation

$$\begin{aligned} \frac{dx}{dt} &= -2x + 2x^2 \\ \frac{dy}{dt} &= -3x + y + 3x^2 \end{aligned}$$

- a) Determine the critical points
- b) State the nature of the critical point
- c) Linearize the system
- d) Solve the system for x and y

QUESTION FOUR (20 MARKS)

Use the Netani's method to solve the following differential equations

- a) $3x^2 dx + 3y^2 dy - (x^3 + y^3 + e^{2z}) dz = 0$ (9 marks)
- b) $2z dz + dy + (2x^2 + 2xy + 2xz^2 + 1) dx = 0$ (11 marks)

QUESTION FIVE (20 MARKS)

Solve the Legendre differential equation $(1-x^2)y'' - 2xy' + p(p+1)y = 0$