MACHAKOS UNIVERSITY

University Examinations 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECOND YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (ELECTRICAL AND ELECTRONICS ENGINEERING)

BACHELOR OF SCIENCE (MECHANICAL ENGINEERING)

BACHELOR OF SCIENCE (CIVIL ENGINEERING)

ECU 203: ENGINEERING MATHEMATICS VIII

DATE: 11/4/2023 **TIME: 11:00 – 1:00 PM**

INSTRUCTIONS:

1) Answer Question ONE and any other TWO questions.
2) Mobile phones and any written material are prohibited in the examination room.
3) No writing should be done on this question paper. Any rough work should be done at the back of the answer booklet and canceled.
4) All answer booklets should be handed in at the end of the exam whether used or not.
5) Programmable calculators are prohibited

QUESTION ONE (COMPULSORY, 30 MARKS)

(a) Obtain a Fourier series of the periodic function $f(x)$ defined by

$$f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$$ (5 marks)

(b)

(i) Expand $f(x) = x^2$ in the range $0 < x < 2\pi$ in a Fourier series. (5marks)

(ii) By letting $x = \pi$ in the series obtained in (i) above, find a series for $\frac{\pi^2}{12}$. (3 marks)

(c) Find the Laplace transform of

(i) $1 + 3t - 2t^2 + \frac{t^6}{12}$ (3 marks)

(ii) $2\sin 4t - 3\cos 2t$ (3 marks)

(d) Use Laplace transforms to solve $4\frac{d^2y}{dt^2} - 12\frac{dy}{dt} + 9y = 0$ given that when $t = 0, y = 2$ and $\frac{dy}{dt} = 4$. (6 marks)

(e) Verify the final value theorem for the function $f(t) = 3 + e^{-t}(\sin t + \cos t)$. (5 marks)

QUESTION TWO (20 MARKS)

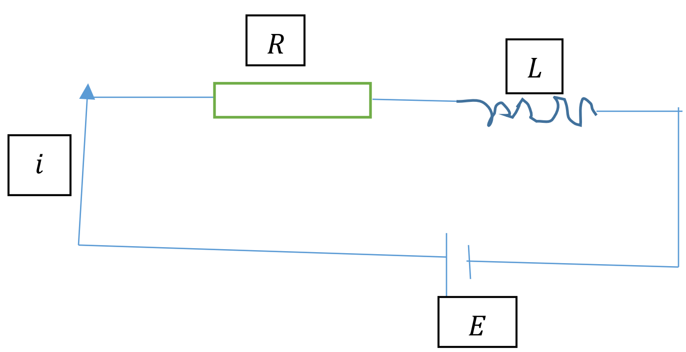
a) Define even and odd functions and hence classify the following functions as either even or odd. (i) $e^x + e^{-x}$ (ii) $\sin x$ (iii) $\cos(2x)$ (iv) $\tan(3x)$. (4 marks)

b) Expand the function $f(\theta) = \theta^2$ as a Fourier series in the range $-\pi < \theta < \pi$ given that $f(\theta + 2\pi) = f(\theta)$. In the series obtained let $\theta = \pi$ and show that $\sum_{n=1}^{\infty}\frac{1}{n^2} = \frac{\pi^2}{6}$. (10 marks)

c) Find the half- range Fourier sine series for the function $f(x) = x$ in the range $0 \le x \le 2$. Sketch the function within and outside of the given range. (6 marks)

QUESTION THREE (20 MARKS)

a) The current i flowing in the circuit shown in the figure below is given by the differential equation $L\frac{di}{dt} + Ri = E$, where E is the constant applied voltage, L the inductance and R the resistance. Using Laplace transforms solve the equation for i given that when $t = 0, i = 0$. (10 marks)



b) Derive Laplace transform of second order derivative and use it to determine $L\{\sin 2t\}$. (10 marks)

QUESTION FOUR (20 MARKS)

a) The value of voltage v volts at different moments in a cycle are given by

θ^0(degrees)	30	60	90	120	150	180	210	240	270	300	330	360
V (volts)	62	35	-38	-64	-63	-52	-28	24	80	96	90	70

Draw the graph of voltage v against angle θ and analyze the voltage into its first three constituent harmonics, each coefficient to 2 decimal places. (12 marks)

b) The voltage from a square wave generator is of the form $v(t) = \begin{cases} 0, -4 < t < 0 \\ 10, 0 < t < 4 \end{cases}$ and has a period of $8ms$. Find the Fourier series for this periodic function. (8 marks)

QUESTION FIVE (20 MARKS)

a) Using time differential property of Fourier transform find the inverse Fourier transform of $x(w) = \frac{jw}{(1+jw)^2}$. (6 marks)

b) Find $L^{-1}\left\{\frac{3s+1}{s^2+2-3}\right\}$. (6 marks)

c) Solve using Laplace transforms $\frac{d^2\theta}{dt^2} - 6\frac{d\theta}{dt}10\theta = 20 - e^{2t}$ given that, $t = 0,\ \theta = 4$ and $\frac{d\theta}{dt} = \frac{25}{2}$. (8 marks)

APPENDIX

Table 1: Elementary Laplace transforms

Function $f(t)$	Laplace transform $L\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$
1	$\frac{1}{s}$
k	$\frac{k}{s}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
t	$\frac{1}{s^2}$
t^2	$\frac{2!}{s^3}$
t^3	$\frac{3!}{s^4}$
t^n(n =positive integer)	$\frac{n!}{s^{n+1}}$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$