MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

THIRD YEAR SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 332: METHODS OF APPLIED MATHEMATICS I

DATE: 16/4/2019 **TIME:**

INSTRUCTION:

Answer Question ONE and any other TWO questions.

QUESTION ONE (COMPULSORY) (30 MARKS)

a) i) State the general second order linear partial differential equation (P.D.E.) (2 marks)

ii) Hence state the three classes of that P.D.E. while giving an example equation of the canonical form in each class. (5 marks)

b) State the Fourier Cosine series expansion of a function $f(x)$ over $[-l, l]$, together with the expressions of the Fourier coefficients. (3 marks)

c) Reduce the heat equation $u_t = \alpha u_{xx}$ to a set of ordinary differential equations. (5 marks)

d) i) Determine the Laplace transform of $e^{at} f(t)$ for some constant a. (2 marks)

ii) Use the Laplace transforms to solve the equation $3y' - 4y = \sin 2t, \quad y(0) = 2$. (5 marks)

e) Given that $u = u(x, y)$, apply the change of variables $\xi = \xi(x, y)$, $\eta = \eta(x, y)$ to express the following in terms of the variables ξ, η.

i) second order partial derivative $u(x, y)$ with respect to y only (4 marks)

ii) second order partial derivative u_{yy} (4 marks)

QUESTION TWO (20 MARKS)

a) Sketch the function $f(x)=\begin{cases}0 & -\pi<x<0\\ 1 & 0<x<\pi\end{cases}$, with $f(x+2\pi)=f(x)$ and hence determine the Fourier series expansion of this function. (9 marks)

b) Classify the partial differential equation $x^2u_{xx}-2xyu_{xy}+y^2u_{yy}=e^x$, and by setting $\eta=x$ transform the P.D.E. to its canonical form. (11 marks)

QUESTION THREE (20 MARKS)

a) Determine the inverse Laplace transform of $\dfrac{2}{(s+1)(s^2+2s+5)}$ using partial fractions

(8 marks)

b) Show that $L\{H(t-a)f(t-a)\}=e^{as}\overline{f(s)}$ where $H(t-a)$ is the Heaviside function

(5 marks)

c) Let the Laplace transform of a function $u=u(x,t)$ be $L\{u(x,t)\}=\int_0^\infty e^{-st}u(x,t)dt$.

Determine the Laplace transform of $\dfrac{\partial^2 u}{\partial t^2}$. (7 marks)

QUESTION FOUR (20 MARKS)

a) Solve $u_t=\alpha u_{xx}$, $0<x<\pi$ subject to: $\begin{array}{l}u(0,t)=u(\pi,t)=0,\ t>0\\ u(x,0)=f(x),\ 0<x<\pi\end{array}$ to obtain the particular solution when $f(x)=x(\pi-x)$. (14 marks)

b) Show that $\{\cos 2x,\sin 2x\}$ is an orthogonal set of functions for $x\in[-\pi,\pi]$. (6 marks)

QUESTION FIVE (20 MARKS)

a) The displacement of a semi-infinite elastic beam is described

$$u_{tt}=a^2u_{xx},\ x>0,t>0$$

by $u(0,t)=f(t)$, $\lim\limits_{x\to\infty}\dfrac{\partial u}{\partial x}=0$, $t>0$. Solve for $u(x,t)$ by the Laplace transform method.

$$u(x,0)=0,\ \left.\frac{\partial u}{\partial t}\right|_{t=0}=0,\ t>0$$

(10 marks)

b) Find the characteristics of the equation $4u_{xx}+12u_{xy}+9u_{yy}-9u=9$ and reduce it to the appropriate canonical form. (10 marks)