MACHAKOS UNIVERSITY

University Examinations for 2022/2023 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS AND STATISTICS

THIRD YEAR ………….. SEMESTER EXAMINATIONS FOR

BACHELOR OF SCIENCE (MATHEMATICS AND COMPUTER SCIENCE)

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

BACHELOR OF SCIENCE (ANALYTICAL CHEMISTRY)

BACHELOR OF SCIENCE (MATHEMATICS)

BACHELOR OF EDUCATION (SCIENCE)

SMA 300: REAL ANALYSIS I

DATE: **TIME:**

INSTRUCTION: *Answer Question **ONE** which is compulsory and any other **TWO***

QUESTION ONE (COMPULSORY)(30 MARKS)

a) For the set $A = (-\infty, 4) \cup \{5,9\} \cup (6,7)$. Find

i) Interior of A

ii) Boundary of A

iii) Exterior of A (5 marks)

b) Explain the following terms

i) Closure of a set

ii) Cauchy sequence.

iii) An upper bound,

iv) Infimum of a set

v) Countable set. (5 marks)

c) Use ratio test to determine the convergence of

$$\sum_{n=2}^{\infty} \frac{(-10)^n}{4^{2n+1}(n+1)}$$ (5 marks)

d) Show that $\lim_{n\to\infty}(1-\frac{1}{2^n}) = 1$ (4 marks)

e) Let A and B be neighborhoods of a point x. Show that the intersection $A \cap B$ is a neighborhood of x (3 marks)

f) Prove that a set $X$is closed iff its complement is open (5 marks)

g) Determine and prove whether the set of all integers is countable. (3 marks)

QUESTION TWO (20 MARKS)

a) If two sets A and $B$are open then prove $A \cap B$ is open (4 marks)

b) Show that if $p \in A$ and $p \notin A'$, then p is an isolated point . (4 marks)

c) Prove that every convergent sequence of real numbers is bounded (4 marks)

d) Apply the integral test to determine the convergence of $\sum_{n=1}^{\infty} \frac{n}{n^2+n}$ (5 marks)

e) Show that the function $f(x) = \begin{cases} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous at 0 (3 marks)

QUESTION THREE (20 MARKS)

a) Show that $\lim_{n\to\infty}(\frac{2x+3}{x+2}) = 1$ (5 marks)

b) Determine the nth partial sum of the series below. Hence test for its convergence.

$$\sum_{n=1}^{\infty} \frac{3}{n^2+3n+2}$$ (5 marks)

c) Show that $f(x) = 2x + 1$ is Riemann integrable on $[1,2]$ and hence find $\int_1^2 (2x+1)dx$, (7 marks)

d) Show that a finite intersection of open sets is open. (3 marks)

QUESTION FOUR (20 MARKS)

a) Define what is meant by a convergent sequence hence show from first principles that the sequence: $y_n = 3 + \frac{(-1)^n}{n^2}$ converges to 3 in $\mathbb{R}$. (5 marks)

b) Given that $A = (3,7) \cup \{10\}$find the exterior and the boundary of A (3 marks)

c) Show that the sequence $\left(\frac{1}{2^n}\right)$ in $\mathbb{R}$ is a Cauchy sequence (4 marks)

d) Use comparison test determine the convergence of

$$\sum_{n=2}^{\infty} \frac{n^2+2}{n^4+5}$$ (4 marks)

e) For each positive integer k, let $\mathbb{R}^k$be the set of all k-tuples where $X = \{x_1, \dots x_k\}$.

Define a function d by $d(x, y) = \sum_{i=1}^{k} |x_i - y_i| \; \forall x, y \in \mathbb{R}$. Show that d is a metric space.

(4 marks)

QUESTION FIVE (20 MARKS)

a) Prove that every convergence sequence is a Cauchy sequence. (5 marks)

b) Prove that the set of rational numbers $\mathbb{Q}$ has no interior point (5 marks)

c) Determine whether the following sequence is monotonic and/or bounded. If bounded determine its supremum and infimum.

$$\left\{\frac{2}{n^2}\right\}_{n=5}^{\infty}$$ (5 marks)

d) Let (S_n) be a sequence of real numbers. If $\lim_{n\to\infty} S_n = l_1$ and $\lim_{n\to\infty} S_n = l_2$ then prove $l_1 = l_2$ (5 marks)