MACHAKOS UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

UNIVERSITY EXAMINATION FOR THE DEGREE OF BACHELOR OF ARTS AND BACHELOR OF EDUCATION

SMA 201: CALCULUS III

DATE:………………………………………………………….. TIME:……………………

INSTRUCTIONS: Answer QUESTION ONE and any other TWO QUESTIONS

QUESTION 1: COMPULSORY (30 MARKS)

a) Consider $I = \int_{x=0}^{1}\int_{y=x}^{\sqrt{x}} xy^2\,dxdy$

i. State the order of integration, giving reasons (2 marks)

ii. Evaluate I (4 marks)

b) Show that $2x^3 + 3x^2y - 2xy^2$ is homogenous function of degree 3 (4 marks)

c) Expand $\ln(1+x)$ as an infinite series (5 marks)

d) Verify Rolle's Theorem for the function $f(x) = (x-2)^6(x-5)^{11}$ in the interval $[2 < x < 5]$ (5 marks)

e) If $f(x) = x^3 - 6x^2 + 11x - 6$ and the interval is $[0 < x < 4]$ find c of the Lagrange mean value Theorem. (5 marks)

f) Using Green's Theorem, evaluate $\int_C [y(2xy-1)dx + x(2xy+1)dy]$ where C the circle is $x^2 + y^2 = 9$. (5 marks)

QUESTION 2 (20 MARKS)

a) If $u = (x, y, z) = x^3 + 2y^3 + 5z^3 + 8xyz$, find $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}$. (3 marks)

b) Find $\int x^4 e^{2x} dx$ (5 marks)

c) Find the minimum value of $x^2 + y^2 + z^2$ when $x + 2y + 4z = 21$ is the constraint. (5 marks)

d) Verify Euler's Theorem on the function $f(x, y) = 2x^3 + 3x^2y - 4y^3$ (7 marks)

QUESTION 3 (20 MARKS)

a) If $u = \tan^{-1}\dfrac{x^3 + y^3}{x - y}$, prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$ (5 marks)

c) Find the $\lim\limits_{x\to 2}\dfrac{x^3+6x^2-36x+40}{x^3-12x+16}$ (5 marks)

c) Expand $f(x) = \sin x$ in the Fourier cosine series in $0 < x < \pi$. (10 marks)

QUESTION 4 (20 MARKS)

a) Expand $\sin x$ as an infinite series (4 marks)

b) Evaluate $\lim\limits_{x\to\frac{\pi}{4}}(\tan x)^{\tan 2x}$ (6 marks)

c) Find the maximum or minimum values of $xy(6 - x - y)$ (10 marks)

QUESTION 5 (20 MARKS)

a) Evaluate the double integral $\iint_R \dfrac{x}{y}\,dxdy$ where R is the rectangle $|x| \le 1,\ 1 \le y \le 2$ (8 marks)

b) Verify Stoke's Theorem for the vector function $F = yi + zj + xk$, for the surface S where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary (12 marks)