



MACHAKOS UNIVERSITY

University Examinations 2018/2019

SCHOOL OF PURE AND APPLIED SCIENCES

DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

FOURTH SECOND SEMESTER EXAMINATION FOR

BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)

SMA 465: PROBABILITY AND MEASURE THEORY

DATE: 7/5/2019

TIME: 2:00 – 4:00 PM

INSTRUCTIONS:

Answer Question One and Any Other Two Questions

QUESTION ONE (COMPULSORY)(30 MARKS)

- a)
- i) State the axioms of a measure, μ .
 - ii) Show that the function $\mathbf{m}$, defined on the the Borel algebra, B , by:
 $\mathbf{m} : (\mathfrak{R}, B) \rightarrow (\mathfrak{R}, B)$ such that $\mathbf{m} \{(a_i, b_i]\} = b_i - a_i$, with a_i, b_i in $\mathfrak{R}$, $a_i \leq b_i$, for $i=1,2,\dots$, is a measure. (6 marks)
- b) Explain, for a given measure space (Ω, Π, μ) ,
- i) Completion of the measure space
 - ii) μ -a.e
 - iii) the Lebesgue-Stieltjes measure space (6 marks)
- c) State:
- i) The Lebesgue Monotone Convergence Theorem. (6 marks)
 - ii) Fatou's Lemma (8 marks)
- d) Prove any one of the results stated in Part (c). (4 marks)

e) Let P be a Probability measure. If A and B are some two events,

- i) Define the conditional probability of A given B .
- ii) Show that the conditional Probability of A given B is a Probability measure.

(8 marks)

QUESTION TWO (20 MARKS)

Let (Ω, Π, p) be a probability measure. Assume $A_j, j=1,2,\dots$ are events in Π .

a) Show that $P^* = \alpha P + (1-\alpha)P, 0 < \alpha < 1$, is a probability measure. (5 marks)

b) If $A_{j+1} \subset A_j, j = 1,2,\dots$

then $P(\bigcap A_n) = \lim_{n \rightarrow \infty} P(A_n)$ (5 marks)

c) If $A_j \subset A_{j+1}, j = 1,2,\dots$

then $P(\bigcup A_n) = \lim_{n \rightarrow \infty} P(A_n)$ (5 marks)

d) $P(A_1 \cup A_2) \leq P(A_1) + P(A_2)$ (5 marks)

QUESTION THREE (20MARKS)

a) Show that for a non-empty set Ω :

- i) The Power set $P(\Omega)$ is a σ -field
- ii) The set $\{\Phi, A\}$ for $A \subset \Omega$ is not a σ -field

(10 marks)

b) Show that if Π is a σ -field then $\bigcap_{i=1}^{\infty} A_i \in \Pi$ for all $A_i \in \Pi, i = 1, 2, 3, \dots$ (10 marks)

QUESTION FOUR (20 MARKS)

a) Consider two measures μ and η defined on a measurable space (Ω, Π) :

- i) Define “ μ is absolutely continuous with respect to η ”
- ii) Explain “ μ -null”
- iii) When is $\mu = \eta$

(9 marks)

