MACHAKOS UNIVERSITY

University Examinations for 2024/2025 Academic Year

SCHOOL OF PURE AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCES

SECOND YEAR FIRST SEMESTER EXAMINATION FOR DEGREE IN BACHELOR OF SCIENCE (TELECOMMUNICATION AND INFORMATION TECHNOLOGY)
BACHELOR OF SCIENCE (MATHEMATICS AND STATISTICS)
BACHELOR OF SCIENCE (STATISTICS AND PROGRAMMING)
BACHELOR OF ECONOMICS AND STATISTICS
BACHELOR OF SCIENCE (MATHEMATICS)
BACHELOF EDUCATION (SCIENCE)
BACHELOR OF EDUCATION (ARTS)
BACHELOR OF ARTS

SMA 202: LINEAR ALGEBRA I

DATE: 21ST JANUARY, 2025 **TIME: 2.00 PM – 4.00 PM**

+INSTRUCTIONS:
Answer question ***ONE (Compulsory)*** *and* ***any other TWO*** *questions*

QUESTION ONE *(COMPULSORY)* (30 MARKS)

(a) If D = $\begin{bmatrix} 4 & -7 & 6 \\ -2 & 4 & 0 \\ 5 & 7 & -4 \end{bmatrix}$ E = $\begin{bmatrix} 3 & 6 & 1/2 \\ 5 & \frac{-2}{3} & 7 \\ -1 & 0 & \frac{3}{5} \end{bmatrix}$ F= $\begin{bmatrix} 3.1 & 2.4 & 6.4 \\ -1.6 & 3.8 & -.19 \\ 5.3 & 3.4 & -4.8 \end{bmatrix}$

Evaluate;
$2D + 3E - 4F$ **(3 Marks)**

(b) Solve the following system of linear equations using Cramer's rule

$$5x_1 + 7x_2 = 3$$
$$2x_1 + 4x_2 = 1$$ **(3 Marks)**

(c) The 3- square matrix A is defined in terms of scalar constant k by $\begin{bmatrix} 2 & -1 & 3 \\ k & 2 & 4 \\ k+2 & 3 & k+7 \end{bmatrix}$.
Given that $|A| = 8$, find the possible values of k. **(5 Marks)**

(d) Find the angle θ between the vectors $\vec{u} = (2,1,3)$ and $\vec{v} = (2,4,5)$. **(4 Marks)**

(e) Let V be a vector space over the field $\mathbb{R}$. Let $S = \{x_1, x_2, ...x_n\}$ be a set of vectors in V. Describe what is meant by:

i) The set S being linearly **dependent**. **(2 Marks)**

ii) The set S being linearly **independent**. **(2 Marks)**

iii) The linear span of S **(2 Marks)**

(f) Verify that the vector $(16,12,-4)$ is a linear combination of $(2,2,0)$ and $(4,2,-2)$ **(2 Marks)**

(g) Demonstrate the linear dependence of the vectors:

$$W = \{(1,1,0),(-1,1,0),(0,1,0)\}$$ **(3 Marks)**

(h) Reduce the matrix given below into its Row-Echelon Normal form and deduce its Rank:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -2 & 0 \end{bmatrix}$$ **(4 Marks)**

QUESTION TWO (20 MARKS)

(a) Let,

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

i) Find A+3B **(2 Marks)**

ii) Find $A^T(BA)$ .Is $A^T(BA)$ symmetric? Justify your answer. **(4 Marks)**

(b) Use the inverse matrix method to solve the following system of linear simultaneous equations

$$S + 2V + 2P = 4$$

$$3S - V + 4P = 25$$

$$3S + 2V - P = -4$$ **(8 Marks)**

(c) Use Cramers rule to solve

$$\begin{aligned} 2x + y - 3z &= 5 \\ 3x - 2y + 2z &= 5 \\ 5x - 3y - z &= 16 \end{aligned}$$

(6 Marks)

QUESTION THREE (20 MARKS)

(a) Let $r = -3i + 2j + k$,
$s = 2i + 4j - 2k$,
$t = 2i + j + 3k$

Find

i) The dot product of the vectors r and t **(2 Marks)**

ii) The cross product of the vectors s and t **(2 Marks)**

(b) Given vectors $\boldsymbol{A} = 3i + 2j - k$; $\boldsymbol{B} = Pi - j + 3k$; $\boldsymbol{C} = i - 2j + 2k$

i) Determine the value of P such that the vectors are coplanar **(5 Marks)**

ii) Determine their vector triple product **(5 Marks)**

(c) Find the basis and the dimension of the subspace of $\Re^4$ given by $(1, -4, -2, 1)$, $(1, -3, -1, 2)$ and $(3, -8, -2, 7)$. **(6 Marks)**

QUESTION FOUR (20 MARKS)

(a) Show that the set $T = \{(1,2,0), (-2,0,1), (0,5,0)\}$ of vectors is a linearly independent. **(3 Marks)**

(b) Given the vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{i} + 6\mathbf{k}$, determine:

i) A unit vector in the direction of the vector $\mathbf{a}$. **(3 Marks)**

ii) The direction cosines and angles of the vector $\mathbf{a}$. **(3 Marks)**

(c) Given the vectors $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - 5\mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$. Determine a unit vector orthogonal to $\mathbf{a}$ and $\mathbf{b}$. **(5 Marks)**

(d) The lines with vector equations:

$\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} + \mathbf{k})$ and

$\mathbf{r} = 5\mathbf{i} + 3\mathbf{j} + 8\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{k})$

intersect. Find the position vector of the point of intersection. **(6 Marks)**

QUESTION FIVE (20 MARKS)

(a) Determine the solution of the system of linear equations below by Gaussian Elimination:

$$\begin{aligned} x - y + z &= 5 \\ 2x - 4y + 3z &= 0 \\ x - 6y + 2z &= 3 \end{aligned}$$

(6 Marks)

(b) Let U and V be a vector subspaces of a vector space W over the field $\mathbb{R}$. Show that the intersection $U \cap V$ is a vector subspace of W. **(4 Marks)**

(c) Let $\mathbf{u}$ and $\mathbf{v}$ be vectors. Prove the dot product $\mathbf{u}.\mathbf{v} = \|\mathbf{u}\|\|\mathbf{v}\|\cos\theta$. **(5 Marks)**

(d) Determine the area of a triangle with the given vertices: $A(1,3,5)$, $B(3,3,0)$, $C(-2,0,5)$. **(5 Marks)**